PHY-421: Mechanics, UMass Amherst, Problem Set #9

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Due: Monday, Nov 9 before 6pm. (Late homework receives 50% credit.)

I. COUPLED OSCILLATIONS

Consider the system below with two degrees of freedom (neglect gravitation). Denote the displacement of each of the particles with respect to equilibrium by $\delta y_i(t)$, $i = 1, 2$.

1. Find the Lagrangian describing the system.
2. Write down the coupled equations of motion for $\delta y_1(t)$ and $\delta y_2(t)$.
3. Find the 2×2 matrices $\hat{T}$ and $\hat{V}$ and solve the normal mode equation for ω: $\det(\hat{V} - \omega^2 \hat{T}) = 0$.
4. Compute the form of the eigenvectors (normal modes), and write down the general solution for $\delta y_i(t)$, $i = 1, 2$ describing the position of each of the particles as a function of time. Your expression should involve 4 constants that are fixed by the initial conditions.
5. Find $\delta y_1(t)$, $\delta y_2(t)$ in the case where the initial velocities are zero $\delta \dot{y}_1(t = 0) = \delta \dot{y}_2(t = 0) = 0$, and the initial displacements are given by $\delta y_1(t = 0) = 0$ and $\delta y_2(t = 0) = d$.