PHY-421: Mechanics, UMass Amherst, Problem Set #8

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Due: Friday, Oct 30. (Warning: Late homework will receive 50% credit).)

I. EQUATION OF MOTION

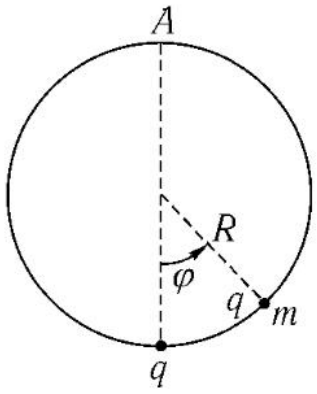
Find the equation of motion of a one-dimensional system described by the following Lagrangian

$$L = \frac{1}{2} e^{\alpha t} \left(\dot{x}^2 - \omega^2 x^2 \right).$$

From the form of the equation of motion, what kind of physical system does this Lagrangian describe?

II. CHARGED PARTICLE AND SMALL OSCILLATIONS

A particle of mass m and charge q moves along a circle in the presence of a vertical gravitational field, see figure. The gravitational field pulls the particle down (parallel to the dashed Aq line). The radius of the circle is R. In the bottom part of the circle there is another charge q, which is fixed (it cannot move and repels the other charge). The potential energy of the moving particle has two contributions: $U(\varphi) = U_{\rm grav}(\varphi) + U_{\rm Coulomb}(\varphi)$, where $U_{\rm grav} = mgh$ (h is the height of the moving particle measured from the bottom of the circle) and $U_{\rm Coulomb} = kq^2/r$ (r is the distance between charges).



1. Assume that φ is small, and write an approximate form of the potential U in term of φ.
2. Find the angle φ_0 corresponding to the equilibrium position (using the approximation of small angle $\varphi_0 \ll 1$ of the previous question).
3. Write the Euler-Lagrange equations for the small oscillations about the equilibrium position $\varphi = \varphi_0 + \delta\varphi$. What is the period of the oscillations?