PHY-421: Mechanics, UMass Amherst, Problem Set #6

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Due: Friday, Oct 16. (Late homework receives 50% credit.)

I. MINIMAL DISTANCE IN SCATTERING

Consider two particles interacting via a repulsive central potential $U(r) = k/r$ with $k > 0$. Find the minimal distance between particles, when one of them (with mass m_1) is coming from infinity with initial velocity v_0, and approaching an initially resting particle (with mass m_2) with impact parameter ρ. The impact parameter is the perpendicular distance between the initial path of an approaching particle and the target particle, see *e.g.* `https://en.wikipedia.org/wiki/Impact_parameter`. Express your result in terms of k, v_0, ρ and the reduced mass.

Hint: Use center of mass coordinates, and focus on the relative motion $\vec{r}$. Use conservation of energy and angular momentum for the relative coordinate: the initial angular momentum should involve the impact parameter ρ. The minimal distance between particles can be determined directly from the effective potential $U_{\text{eff}}(r)$ and the energy E.

II. BOUNDED MOTION

A particle moves in a central potential $U(r) = -\frac{k}{r}\mathrm{e}^{-r/\xi}$ (screened potential, with $k > 0$ and $\xi > 0$). For what values of the angular momentum ℓ_0 a bounded motion of the particle (i.e. when the particle does not escape to the infinity) is possible?

Hint: A bounded motion is possible if the effective potential has a minimum – you can admit that if the effective potential has an extremum, then this is a minimum. You might find it useful to introduce the dimensionless variable $u = r/\xi$, and to study the function $f(u) = u(1+u)\mathrm{e}^{-u}$.