PHY-421: Mechanics, UMass Amherst, Problem Set #5

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Due: Friday, Oct 2 before 5pm. (Late homework receives 50% credit.)

I. TRAJECTORY

Let us throw a particle of mass m from the north pole with a velocity $\vec{v}_0$ forming an angle α with the surface of the earth. Considering only the effect of gravity and neglecting air friction, compute the value of v_0 so that the particle will land precisely on the south pole. Express your result in terms of α, G (Gravitational constant), M the mass of the earth and R its radius. What is the eccentricity of the trajectory? (in terms of α only)

Hint: Use the equation of the trajectory $r(\theta)$ derived in class, and evaluate the initial angular momentum ℓ_0. What is $r(\theta = \pm\pi/2)$? To compute the eccentricity, use the formula $\epsilon = \sqrt{1 + \frac{2E\ell_0^2}{mk^2}}$ from the lecture notes.

II. ANOTHER CONSTANT OF MOTION FOR THE KEPLER PROBLEM

Show that the vector $\vec{C} = \vec{p} \times \vec{\ell} - mk\vec{e}_r$ is a conserved quantity for the motion in a central potential $U(r) = -k/r$. Here $\vec{p}$ is the linear momentum, and $\vec{\ell}$ the angular momentum.

III. THREE DIMENSIONAL HARMONIC POTENTIAL

Study the motion of a particle of mass m in a central potential $U(r) = \gamma r^2$ with $\gamma > 0$.

Hint: Write the equations of motion in Cartesian coordinates x, y, z and notice that they are decoupled.