PHY-421: Mechanics, UMass Amherst, Problem Set #4

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Due: Friday, Sept 25. (Late homework receives 50% credit.)

I. SPRING, EQUILIBRIUM POSITIONS AND SMALL OSCILLATIONS

A particle of mass m is restricted to move without friction on the x axis. This particle is connected to a fixed point P a distance d from the x axis by a spring with constant k and length at rest ℓ_0. Working in the two dimensional plane (x, y), we will denote the coordinates of the particle by $(x, 0)$ while the point P has coordinates $(0, d)$. Find the equilibrium positions of the particle and discuss their stability. In the case where the equilibrium positions are stable, find the period of the small oscillations near equilibrium.

Hint: You will want to distinguish the cases $d > \ell_0$, and $d < \ell_0$ (ignore the special case $d = \ell_0$).

II. OSCILLATIONS OF A FLOATING OBJECT

Consider a cubic object of mass $M = V\rho_{\text{solid}}$ and volume $V = a^3$ that is floating and stationary in a liquid of density ρ, such that a volume V_s of the object is submerged. Compute the period of the oscillations if we displace the object by a small amount (in the vertical direction) from its equilibrium position.

Hint: Recall Archimedes' principle: the buoyant force is equal to the weight of the volume of liquid displaced.

III. TRIANGLE WAVE FORCED OSCILLATOR (NOT GRADED)

Consider a damped oscillator subject to a periodic triangle wave force $F(t)$ with period T, such that $F(t+T) = F(t)$, $F(t) = F_0(-1 + 4t/T)$ for $0 \leq t \leq T/2$ and $F(t) = F_0(1 - 4(t - T/2)/T)$ for $T/2 \leq t \leq T$. Assume the oscillator is underdamped. Compute the response of the oscillator forced by this $F(t)$ using Fourier series.

Though the integrals in this problem are readily computed by hand, feel free to use Mathematica or equivalent to evaluate them.