PHY-412: Mechanics, UMass Amherst, Problem Set #1

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Due: Friday, Sept 4 by the end of the day. (Late homework receives 50% credit.)

I. LEVI-CIVITA SYMBOL

Consider the completely antisymmetric Levi-Civita symbol ϵ_{ijk}, where all indices i, j, k run from 1 to 3 and $\epsilon_{123} = 1$.

1. Show that

$$\sum_{j,k=1}^{3} \epsilon_{ijk}\epsilon_{ljk} = 2\delta_{il}.$$

hint: show that the right-hand side and the left-hand side of this equation are equal, for all values of i and l. Consider the cases $i = l$ and $i \neq l$ separately.

2. Using the formula $\sum_{k=1}^{3} \epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$, prove the following identity

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}.$$

3. What is the value of

$$\sum_{j,k=1}^{3} \epsilon_{ijk}g_{jk},$$

with g_{jk} symmetric such that $g_{jk} = g_{kj}$.

II. TRAJECTORY

A particle moves along a helicoidal trajectory around the z axis, with cylindrical coordinates:

$$r = R,\ \theta = \omega t,\ z = H\left(1 - \frac{\theta}{2\pi}\right),$$

where R, ω, H are positive constants. The particle starts at $t = 0$, and goes around the z axis before ending the trajectory on the $z = 0$ plane.

1. Compute the velocity $\vec{v}$ of the particle (in cylindrical coordinates). Sketch the trajectory.
2. Compute the acceleration $\vec{a}$ of the particle. What is $\vec{a}.\vec{v}$?
3. What is the length of the trajectory?

III. POLAR COORDINATES

Derive the expression of the velocity $\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$ and acceleration $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e}_\theta$ in polar coordinates $(r(t), \theta(t))$.