A Prelude to Quantum Field Theory
Problems for Chapter 7

1) Missing interactions For this problem we start with a theory two real fields $\phi$ and $\chi$ with the Lagrangian $L_I = -g\chi^2\phi^2$.

a) Write out the Feynman rule for this theory.

b) There is no tree level scattering amplitude for $\chi + \chi \to \chi + \chi$. Draw the one-loop diagrams for this process, and write them as a Feynman integral.

c) Now you can evaluate that loop integral, using the results given in Eq. (7.20) or (7.21). Do this.

d) This appears to lead to a problem - there is a divergence in the result. Unlike the example used in the text, this cannot be absorbed in the renormalization of the original interaction. What has gone wrong? The problem is that we were not general enough in defining the theory. With real scalars we should have also included interactions $L_I = \lambda_1 \chi^4 + \lambda_2 \phi^4$. Show how one can renormalize these interactions and how that hides the divergence found in part c). This highlights a general lesson. All renormalizable interactions which are allowed by the symmetries of a theory should be included in the Lagrangian.

2) A one loop calculation. Consider a scalar field with a cubic selfinteraction with Lagrangian

$$L = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{3} \phi^3.$$ 

Compute the one-loop counterterms, as a function of the physical mass $m$ and the physical coupling $\lambda$, in dimensional regularization, for the mass and the wave function of $\phi$. 