A Prelude to Quantum Field Theory
Problems for Chapter 6

1) **Relativistic Coulomb scattering** Compute the full, relativistic expression for $d\sigma/d\Omega$ to $O(e^4)$ for Coulomb scattering, generalizing Eq. 6.31.

2) **Yukawa interaction** Calculate the potential $V(r)$ between two nearly static charges for the massive vector particle of Eq. 6.60. You will find that it is exponentially suppressed at large distances. Identify the screening length $\xi$ from the exponential factor $e^{-r/\xi}$.

3) **A cross section.** Consider the theory of two complex scalar fields $\phi$ and $\chi$ with masses $m_\phi$ and $m_\chi$, both carrying electromagnetic charge $q$.

   1. Write down the Feynman rules for the vertices that describe the theory.
   2. Compute, to leading order in $q$, the matrix element for the scattering $\phi^+\phi^- \rightarrow \chi^+\chi^-$.
   3. Use this matrix element to compute the total cross section of the process.

4) **A decay rate** We can now do a complete calculation of a realistic system. A simple yet physical example is the decay $\Sigma^+ \rightarrow P\pi^0$. Here the $\Sigma^+$ is a spin 1/2 particle similar to the proton $P$, but with a mass of 1189 MeV/c$^2$ compared to the proton’s mass of 938 MeV/c$^2$. For this problem the $\pi^0$ can be treated as a real scalar field with a mass of 135 MeV/c$^2$. (A book on particle physics will tell you that it is a pseudo-scalar and will explain what that means.) If we were to write a Lagrangian to describe this transition we could use non-relativistic fermion fields for the spin 1/2 particles, such that

   $$\mathcal{L}_I = -g\psi_P^*\psi_\Sigma\phi_\pi$$  \hspace{1cm} (1)

   a) Use dimensional analysis to show that the coupling constant $g$ is dimensionless in natural units, much like the electric charge $e$.

   a) Use this to calculate the decay rate.
b) The $\Sigma^+$ lifetime is $\tau = 1/\Gamma = 0.8 \times 10^{-10}$ sec, and this mode has a branching fraction ($=\Gamma_{P\pi}/\Gamma_{tot}$) of 52%. Use this information to calculate the coupling strength $g$. You should find a number very much smaller than the electric charge. This is an indication that the decay is due to the weak interaction, which is, after all, weak.