**Math 421** **Problem Set 5** **Fall 2010**

## Due: Friday, Oct. 22

*Note:* Don't panic—some of these are quite quick to do!

1. Do page 78, Exercise 2 (b) and (d). The domain of each function is understood to consist of all complex $z$ for which the quotient makes sense.

2. Do page 78, Exercise 10.

3. (a) Do page 84, Exercise 6 (a). [All you need to do is to show that $\big(f_2(x)\big)^3 = z$ for all z in the domain of $f_2$.]

   (c) Do page 84, Exercise 6 (c).

4. Do page 98, Exercise 1 (b). You may use derivative rules (3-4)–(3-10) on page 96.

5. Do page 98, Exercise 2 (b).

6. Do page 99, Exercise 11. Note that you have to show two things: first, $f$ *is* differentiable at $z_0 = 0$; second, $f$ is *not* differentiable at $z_0 \neq 0$.

   (*Hint:* There is a "sneaky" way to do the latter that's a lot easier than the way the book suggests!)

   *Note:* This function is thus differentiable at the point $z_0 = 0$ but *not* holomorphic there.

7. Do page 110, Exercise 1 (d).

8. Do page 111, Exercise 8 (a).

9. Do page 112, Exercise 14.

10. Find explicit formulas for the stereographic projection $p\colon \Omega \to \widehat{\mathbb{C}}$ and its inverse $q\colon \widehat{\mathbb{C}} \to \Omega$.

    Here $\Omega = \{\, (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1 \,\}$ is the Riemann sphere and $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ is the extended complex plane.

    Of course $q(\infty) = (0, 0, 1)$, the North pole and $p(0, 0, 1) = \infty$, so all you need to do is find formulas for $q(z)$ when $z \in \mathbb{C}$ and for $p(u, v, w)$ when $(u, v, w) \in \Omega$ with $w \neq 1$.