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Is thirty-two three tens and two ones? The embedded structure of cardinal numbers

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ABSTRACT

The acquisition and representation of natural numbers have been a central topic in cognitive science. However, a key question in this topic about how humans acquire the capacity to understand that numbers make 'infinite use of finite means' (or that numbers are generative) has been left unanswered. While previous theories rely on the idea of the successor principle, we propose an alternative hypothesis that children's understanding of the syntactic rules for building complex numerals—or numerical syntax—is a crucial foundation for the acquisition of number concepts. In two independent studies, we assessed children's understanding of numerical syntax by probing their knowledge about the embedded structure of cardinal numbers using a novel task called Give-a-number Base-10 (Give-N10). In Give-N10, children were asked to give a large number of items (e.g., 32 items) from a pool that is organized in sets of ten items. Children's knowledge about the embedded structure of numbers (e.g., knowing that thirty-two items are composed of three tens and two ones) was assessed from their ability to use those sets. Study 1 tested English-speaking 4- to 10-year-olds and revealed that children's understanding of the embedded structure of numbers emerges relatively late in development (several months into kindergarten), beyond when they are capable of making a semantic induction over a local sequence of numbers. Moreover, performance in Give-N10 was predicted by other task measures that assessed children's knowledge about the syntactic rules that govern numerals (such as counting fluency), demonstrating the validity of the measure. In Study 2, this association was tested again in monolingual Korean kindergarteners (5–6 years), as we aimed to test the same effect in a language with a highly regular numeral system. It replicated the association between Give-N10 performance and counting fluency, and it also demonstrated that Korean-speaking children understand the embedded structure of cardinal numbers earlier in the acquisition path than English-speaking peers, suggesting that regularity in numerical syntax facilitates the acquisition of generative properties of numbers. Based on these observations and our theoretical analysis of the literature, we propose that the syntax for building complex numerals, not the successor principle, represents a structural platform for numerical thinking in young children.

1. Introduction

The mental representation of natural numbers has been studied for decades, with children's acquisition of number concepts at the center of scientific inquiry. The acquisition of number concepts—notably the meaning of number words and the generative properties of natural numbers[2]—is not trivial. It takes years for a developing child to understand the meaning of numbers and their relations (Carey, 2004), and

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[2] A set of logical rules that allows the generation of new numbers indefinitely. In case of cardinal numbers (i.e., verbal numbers denoting cardinality), Merge and labeling operations allow the building of hierarchical structure of numerals indefinitely (see Section 1.2). In natural numbers in their abstract form, the first and second order Peano's Axioms define the model and its properties to build the natural numbers indefinitely.

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the concept of precise numerical value may never be nor never need to be fully acquired for numerical reasoning to be used (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). Most, if not all, psychological theories assert that the acquisition of number concepts culminates when children understand the successor principle[3] that refers to the recursive rule according to which any natural number has a successor that is also a natural number. In this paper, we will argue that the successor principle is not the ideal nor the full representation for explaining the cognitive mechanism by which children conceptualize number. Instead, we develop alternative forms of mental representations for natural numbers based on the linguistic structure of numerals which we believe capture psychological reality.

1.1. Theories about children's acquisition of the successor principle

An earlier theory about children's acquisition of number concepts asserts that the symbolic number system, represented by number words organized in a counting sequence, share the same abstract structure with the approximate number system, a nonverbal mechanism for representing approximate numerical quantities, and that the isomorphism between the two systems drives children's acquisition of the meaning of number words (Dehaene, 2001; Gallistel & Gelman, 1992, 2005). Some of these authors further argue for an innate nonlinguistic recursive mechanism, the integer symbol generator, that generates mental symbols to represent discrete quantities which calibrates approximate values of the approximate number system to the precise values represented by number words (Leslie, Gelman, & Gallistel, 2008). According to Leslie et al. (2008), this mechanism includes a representation for the symbol *one* and the recursive rule of succession. At least two developmental predictions can be drawn from this integer symbol generator theory: 1) If this integer generator includes the representation of one, then children should acquire the concept of the cardinal number one without difficulties; 2) if the successor rule is embedded in the integer generator, children should be able to deduce the next cardinal value to any number, potentially infinitely. There is evidence against both predictions. First, children usually spend two or three years acquiring the cardinal number one, after which they spend another 6 to 9 months understanding the cardinal number two (Wynn, 1990, 1992). These results suggest that children do not have early access to the cardinal meaning of one. Second, recent studies show that children who understand the counting principles do not know what the next cardinal number is even for small numbers within their counting range (Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018). Additionally, cross-linguistic analyses show that some natural languages use words to represent sets of one, two, and three but lack symbols to represent higher quantities (Corbett, 2000). Such findings suggest that the knowledge of the next number is available neither to all children nor all languages.

Carey (2004) states that the meaning of number words is acquired through the interaction between a perceptual/attentional mechanism for representing a small number of objects (the so-called object file system) and children's exposure to utterances of quantifiers, plural and singular expressions, and the counting sequence (Le Corre & Carey, 2007, 2008). According to this account, the successor rule is acquired by an inductive inference in which children realize that an increment of one item in the environment or in working memory is equivalent to moving one position in the counting sequence. Children at first apply this rule gradually in a smaller set size (up to four or five items) but become able to generalize the rule, at least within their count list, in a state known as cardinality principle (CP) knowledge (Wynn, 1992). A study by Sarnecka and Carey (2008) indeed demonstrated that only CP knowers reliably understand that adding exactly one item means moving exactly one position forward in the counting sequence, evaluated by the so-called Unit task. More recent studies, however, have shown results inconsistent with that previous conclusion, as many CP knowers were found to fail the Unit task even within the range of their count list (Cheung, Rubenson, & Barner, 2017; Davidson, Eng, & Barner, 2012; Spaepen et al., 2018). These newer data thus question the theoretical argument that CP knowledge is derived from the acquisition of the successor rule. It should be noted, however, that the interpretation of children's performance in the Unit task requires caution because, in theory, this task can be solved successfully as long as a child has a local successive knowledge of cardinal numbers between one and nine. Thus, this task may not inform anything about the child's knowledge about the successor, or the next number, in any abstract sense. We return to this point later.

More recently, the idea of the successor principle has been investigated from the perspective of infinity and its relations to counting sequences. For example, Cheung et al. (2017) found that only CP knowers who can count up to a high number (close to one hundred) without an error have an explicit knowledge of infinity. These results support the view that understanding the structure of the counting sequence is an important step towards acquiring the concept of infinity; however, they do not provide a mechanistic explanations for the acquisition process.

1.2. The syntactic rules of complex numerals

A potentially effective approach to studying the mental representation of generative properties of natural numbers is investigating the syntactic rules that form complex numerals. After all, natural numbers are expressed in numerals, and children acquire the meaning of natural numbers by learning number words in a base number system (i.e., not by abstract rules such as $S(n) = n + 1$ in the Peano's axioms). This base system represents numbers using an arithmetic algorithm which defines the cardinal value using the operations of addition and multiplication (i.e., 206 is equivalent to $2 \times 10^2 + 0 \times 10^1 + 6 \times 10^0$) but does not require the use of the successor function.

Hurford (1975, 1987) in his seminal work first formalized the idea that recursion in number is enabled by the syntactic rules of complex numerals. He states that cardinal numbers are divided into two types: lexical primitives and complex numerals. The former category consists of unique words that must be individually memorized (e.g., the numbers from one to nine and some multipliers). The latter are the product of the combination of the lexical primitives using Merge, a fundamental linguistic operation in charge of building linguistic structures (Chomsky, 2014; Ohta, Fukui, & Sakai, 2013). In most languages, complex numerals are represented using additive and multiplicative bases (most typically in a power of ten or base-10) and share a set of phrase structure rules which, for simplicity, we refer to as *numerical syntax* in the rest of this paper. According to Hurford (2007), these rules operate on the lexical classes of Digits (e.g., one, two, …, nine) and Multipliers (e.g., -ty, hundred, thousand; also referred to as M), resulting in the lexical category of either Number or Phrase. A Phrase is either a bare Multiplier (e.g., mil for 1000 in Spanish) or a result of a Merge between a Number and a Multiplier as in two hundred for 200 in English. A Number is a bare Digit (e.g., Six), a Phrase (e.g., [Two Hundred]), or a Merge of a Phrase and a Number (e.g., [[Two Hundred] [Six]]).[4] These categories have a formal arithmetic interpretation that

[3] In the field of numerical cognition, the successor function and its properties are usually defined as a summary of the first and second order Peano's Axioms. The basic statement is that there is a function S whose input (n) and output are natural numbers and that the relation between input and output is the operation $n + 1$. Importantly, this relation is applied to any natural number; ergo every natural number has a successor, which leads to infinity.

[4] This analysis of numerical syntax is only partially articulated from the perspective of modern linguistic theories, for example as can be seen in the

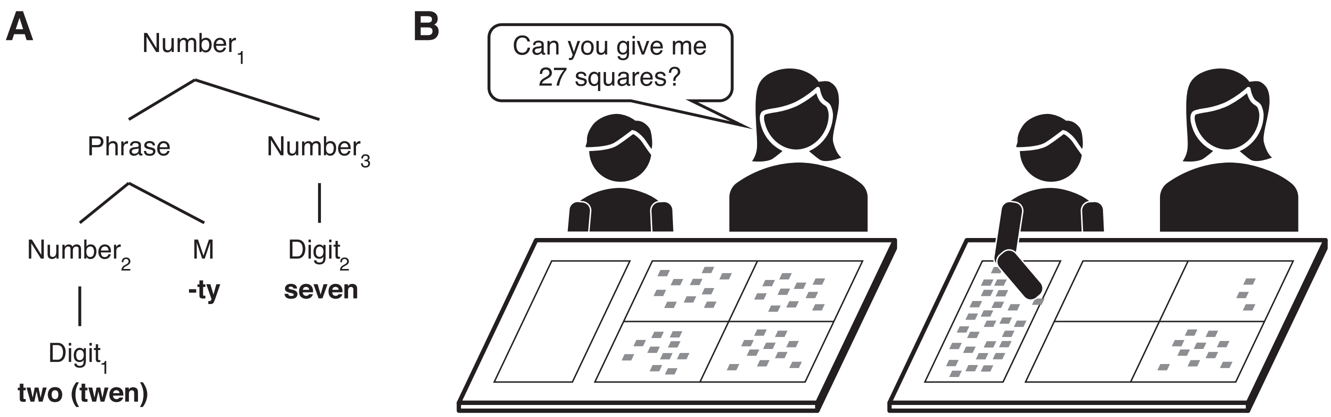

Fig. 1. A graphical illustration of numerical syntax and the Give-N10 task. A. Syntax tree for the numeral twenty-seven according to the phrase structure rule developed by Hurford (1975). Note that Number2 and Number3 are recursively embedded in Number1. B. In the Give-a-number Base-10 task (Give-N10), the participant is given four sets of 10 squares and is asked to give, for example, 16, 27, and 32 squares. Importantly, the experimenter models a trial of acquiring a large number of squares maximally using the base-10 knowledge.

reflects the additive and multiplicative operations in a base number system. The Merge of a Number and a Multiplier represents a multiplicative composition (e.g., [$_{Phrase}$ [$_{Number}$ [$_{Digit}$ Two]] [$_{M}$ Hundred]] for 2×10^2) and the Merge of a Phrase and a Number represents an additive composition (e.g., [$_{Number}$ [$_{Phrase}$ Two Hundred] [$_{Number}$ [$_{Digit}$ Six]] for 200 + 6). According to Hurford (2007), one of the most important features of this system is that the category Number is recursive. That is, a Number can be embedded in another Number either directly underneath or via Phrase (see Fig. 1A). Under this theoretical framework, the generative properties of natural numbers are enabled by the recursive rules of numerical syntax and not by the knowledge of the succeeding number, making the successor principle unnecessary.

There are variations in how developed numeral systems reflect arithmetic operations, even within the base-10 number system. Some numeral systems are considered more regular because they clearly reflect the additive and multiplicative operations. In Korean, the numbers 12 and 20 are represented as *sib-ee* (i.e., ten-two) and *ee-sib* (i.e., two-ten) in which the same digit and multiplier are merged in two different ways: Phrase-Number (i.e., that represents the additive composition 10 + 2) and Number-Multiplier (i.e., that represents the multiplicative composition 2×10), respectively. In contrast, English cardinal numbers twelve and twenty are lexically non-transparent. The influence of the transparency of cardinal numbers in their production has been shown in cross-linguistic and longitudinal studies. Miller and Stigler (1987) found that Chinese children count significantly higher than American children of the same age. Lefevre et al. (2002) also showed that Canadian English-speaking children master the counting sequence before their French-speaking peers, attributing results to greater irregularities in the French numeral system. Leybaert & Van Cutsem (2002) compared an additive base-5 system used by deaf Belgian children and a base-10 system used by hearing Belgian children and found that deaf children commit more errors when counting to the number 6, which is the first position where the additive rule applies in Belgium sign language. Gould (2017) found that Australian kindergarteners show difficulties in abstract counting beyond certain numbers, including twelve, thirteen, twenty-nine, thirty-nine, and one hundred nine. In English, these “stopping points” represent the cardinal numbers prior to the Phrase that represents addition (i.e., twelve, thir[teen], four[teen]), prior to the Phrases that represent multiplication (i.e., thir[ty] and for[ty]), or the first Phrase-Number in which the Number category is a bare Multiplier (i.e., one hundred ten). These studies suggest that syntactic regularities influence the acquisition of the rules to correctly produce the counting sequence. Nevertheless, they do not address the conditions in which syntactic rules are acquired nor the relationship between the syntactical knowledge and the conceptual representation of cardinal values.

(*footnote continued*)
unique use of the notion Phrase in this analysis.

1.3. The present study

As a step towards understanding children's acquisition of the generative properties of natural numbers, we investigated children's cardinal understanding[5] of numerical syntax using two independent studies in two languages (English and Korean). We developed a novel task to assess children's understanding of the embedded structure of cardinal numbers. We then examined the age at which children acquire this knowledge and tested how this knowledge relates to other types of numerical knowledge. This novel task (which we call Give-a-Number Base-10 or Give-N10) is much like the well-known Give-a-Number task (Wynn, 1990), as children are asked to give a specific number of items to the experimenter. Critically, however, the task utilizes four spatially distinct groups of ten items, and the children are asked to give a large number of items expressed in complex numerals such as sixteen, twenty-seven, and thirty-two (see Fig. 1B). Importantly, prior to testing, the experimenter models a case of retrieving 31 items by grabbing three groups of ten items one at a time and then adding a single item from the last group, illustrating the syntactic nature of the numeral thirty-one, [[three -ty] one]. We reasoned that if children understand the syntactic structure of cardinal numbers and its meaning, they would produce the inquired number maximally using the groups of ten items.

We hypothesized that this cardinal understanding of the embedded structure in complex numerals is one of the last stages towards children's understanding of generative number concepts yet is founded on children's earlier-developing abilities to process syntactic rules of complex numerals (such as rote counting). To test these hypotheses, we administered several other tasks. The *Unit task* (Sarnecka & Carey, 2008), which claims to test children's understanding of the successor principle, was administered to test whether this “successor principle”

[5] We use the term *cardinal* to refer to the knowledge about the number of elements in a set. This is to be distinguished from a purely verbal or syntactic knowledge by which a child can rote count but may not have an understanding of the meaning of that number.

knowledge comes earlier than the understanding of the embedded structure of cardinal numbers. Two other tasks, *Interval-Counting* and *Number-Multiplier-Syntax* evaluated children's knowledge about the syntactic rules that govern numerals, which allowed us to test the validity of the novel Give-N10 task and to situate children's Give-N10 performance in the context of other numerical knowledge.

2. Study 1

2.1. Methods

2.1.1. Participants

Sixty-six English-speaking children participated in the study (mean age = 6 years 3 months; range = 4;5–9;8). The distribution across school grade was as follows: nine preschoolers, twenty-four kindergarteners, eighteen 1st graders, five 2nd graders, eight 3rd graders, and two 4th graders. The study spanned over multiple months into a school year, thus children's grade, which we used in our analysis, was adjusted by the date of children's participation in the study. For example, a child tested in September was given an additional 1/12 of a grade and a child tested in February was given an additional 6/12 of a school grade. All were fluent native English speakers and had no history of developmental disability. Participants were recruited from the Western Massachusetts area and were tested at the University of Massachusetts Amherst in the Cognitive and Developmental Neuroscience laboratory. The majority of the children belonged to middle-class families. Fifty-eight children were identified as White or Caucasian, three children as Asian, two children as Hispanic or Latino, one as Native Hawaiian, and the other two did not provide information. Children received stickers and a small gift for their participation, and the family received $10 for travel compensation. Three additional children were originally enrolled in the study but were not included in data analysis: two of the three did not complete the study and the third had a very short count list (being able to count only up to 18). All procedures were approved by the University of Massachusetts Institutional Review Board.

2.1.2. Procedure and task

Children played the following five interactive games/tasks with the experimenter. The *Give-a-Number* task (Cheung et al., 2017; Wynn, 1990, 1992) and the *Unit task* (Sarnecka & Carey, 2008) evaluated children's understanding of the cardinality principle and sequential cardinal knowledge, respectively. The *Interval-Counting* task and the *Number-Multiplier-Syntax* task evaluated children's knowledge about the syntactic rules that govern numerals. Finally, the *Give-a-Number Base-10* task evaluated children's cardinal knowledge about the syntactic structure of complex numerals. The five tasks were tested in the following order in all children: Interval-Counting, Unit, Give-a-Number Base-10, Number-Multiplier-Syntax, and Give-a-Number. The entire procedure took approximately 40 min.

2.1.2.1. Interval-Counting. This task evaluated children's knowledge of the abstract counting sequence, similar to what was used in previous studies (Davidson et al., 2012; Gervasoni, 2003; Gould, 2017; Miller & Stigler, 1987; Siegler & Robinson, 1982; Song & Ginsburg, 1988). Unlike previous studies, however, we asked children to count within given intervals: from 15 to 24, from 37 to 51, and from 95 to 111. For example, for the second interval the experimenter asked "Can you count from 37 for me?" If the child failed to respond, the experimenter encouraged the child by saying the three consecutive numbers that come before the starting number. In the above example, the experimenter would say "Let's count together: 34, 35, 36" with a rising intonation to encourage the child to continue counting alone. When the child reached the upper bound of the interval, say 51, the experimenter stopped the child and said "Thank you." We used intervals because many children found reciting the entire count list (from 1) tedious and boring. The three intervals were chosen based on previous observations (e.g., Gould, 2017), which demonstrated that children show difficulties counting particularly at decade boundaries, for example, going from 29 to 30, from 39 to 40, and from 99 to 100.

In order to quantify children's knowledge of the regularities in the counting sequence, the Decades Transition (DT) score was defined from this Interval-Counting task. Each child's DT score was computed as the number of times that the child passed the decade boundaries. The points of decade transition were 19, 39, 49, 99, 109. Each transition was marked 0 (fail) or 1, (pass) making a maximum of 5 points across all the intervals. One prevalent idea is that children learn the count list progressively (e.g., they learn the 39-to-40 transition before the 49-to-50 transition). Consistent with this view, all of our child participants who passed the higher decade passed the lower decade also, with only four exceptions (three children passed 99 but not 49, and one child passed 99 but not 39). Thus, we reasoned that overall competence to produce decade names can be unbiasedly quantified if each decade transition is treated as an independent item in a test.

2.1.2.2. Unit. This task was originally developed by Sarnecka and Carey (2008) and was later adopted by other authors including Davidson et al. (2012) and Cheung et al. (2017), who call it the successor task. For Sarnecka and Carey (2008), this task allows to infer children's acquisition of the successor function by testing how children associate the next number in the count list with the next cardinal value of a set. While the original version, used by Sarnecka and Carey (2008), used numbers between 4 and 7, later versions by Cheung et al. (2017) used a higher range up to 78. We followed these later versions and used the following eight numbers: 5, 6, 12, 17, 24, 36, 57, and 75. The materials used in this task were an opaque plastic cylinder and small plastic bears. At the beginning of the task the experimenter said, "Now, we are going to play with this container by putting bears inside" to the child. On each trial, the experimenter presented and guided the child's attention to the empty container and said, "I am going to put [N] bears in the container," where N was equal to the number of objects tested on the trial. To ensure that the child remembered the number of items in the container, the experimenter asked, "How many bears are there in the container?" If the child answered incorrectly or failed to respond, the experimenter said, "Oops, let's try again! Remember, I put [N] bears in the container." The trial was repeated until the child responded correctly to this memory-check question. Our protocol specified to skip a number if the child gave a wrong answer in the memory-check questions three times; however, none of the children failed to repeat the correct number. After correctly answering the memory-check question, the experimenter said, "Right! Now watch." Simultaneously, the experimenter added one bear to the container and elicited a choice between N + 1 and N + 2 by saying, "Are there [N + 1] bears or [N + 2] bears in the container?" The order of the numbers (i.e., N + 1 and N + 2) in the questions was counterbalanced within and between children. This task consisted of eight, two-alternative forced-choice trials. The accuracy that corresponds to the number of correct answers divided by the total number of trials was computed for each child's performance measure.

2.1.2.3. Give-a-Number Base-10 (Give-N10). This task evaluated children's cardinal knowledge of the syntactic structure of complex numerals, especially the comprehension of the additive and multiplicative operations in a base-10 number system. Simply put, it tested whether children understand that, say, a cardinal number "twenty-seven" is composed of two tens and a seven (Fig. 1A). The materials for this task were 40 blue squares, each with 1 inch sides, and a template of polished clear vinyl with five rectangular areas. Four of these areas were used by the researcher to locate sets of squares, and the fifth was used for the child to give the answer (Fig. 1B). At the beginning of the task, the experimenter said to the child, "Let's play a new game with the squares now. Every time I'm going to ask you to give me some squares. All right, let's play!" The researcher presented 40 individual squares organized in 4 sets of 10 squares to the child and said, “Look, I have 10 squares here.” The researcher then counted out one set of ten squares out loud with the child. After counting, the researcher pointed to each of the remaining three sets and said, “There are also 10 squares here, 10 squares here, and 10 squares here.” The experimenter proceeded to model a trial of acquiring 31 squares by using sets of 10 squares representing the base-10 system, “Now, I am going to take 31 squares. Look, ten, twenty, thirty, thirty-one.” While saying this, the researcher first took three sets of ten squares set by set and added one individual square. The experimenter modeled this activity of retrieving squares using base-10 knowledge in order to facilitate the use of that sophisticated strategy, inspired by a previous study which showed that children easily adopt a modeled strategy when they have the conceptual knowledge for using it (Miura, Okamoto, Kim, Steere, & Fayol, 1993). After this modeling, the task started when the experimenter asked, “Can you please give me N squares?” Each child was asked to give the following six numbers in that order: N = 3, 7, 10, 16, 27 and 32.

The performance and success in this task were analyzed based on the pattern of children's response to the numbers 16, 27, and 32. Children earned 0, 0.5, or 1 point for each of the three trials. The following two criteria were applied for a child to receive the score of 1. First, the child must retrieve the correct number of squares. Second, the child must use the strategy of retrieving the sets of 10 squares and maximize the use of that strategy. That is, to give 16 squares, the child must retrieve one set of ten squares; for 27 squares, the child must retrieve two sets of ten squares; and for 32 tiles, the child must retrieve three sets of ten squares. If a child used at least one set of 10 squares but additionally retrieved more than 10 squares without using sets of 10, the child received the score of 0.5. Cases other than these patterns were given the score of 0.

2.1.2.4. Number-Multiplier Syntax. This task evaluated children's knowledge about the syntactical structure of verbal numbers. An open abacus with three columns was used in this task. The three columns each represented the ones, tens, and hundreds unit (i.e., Multipliers) in that specific order. Round beads were used to represent the number in each of the units (i.e., Digits). Children were tested on their knowledge about how the number of beads in each of the three units all together make a whole number. No beads were present in the abacus at the beginning of each trial, and only the experimenter added or removed the beads from it. There was a total of eight trials, four of which tested the number in the tens and the other four tested the number in the hundreds.

At the beginning of the task, the researcher said, while pointing to an open abacus, “We can use this toy to make numbers. I am going to show you how it works. If I put one bead here, we have the number one. If I put another bead here, we have the number two.” The researcher put the beads in the rightmost column (ones unit) of the abacus and asked, “Can you please make the number three for me?” After the child gave the correct answer, the researcher retrieved the three beads to empty the abacus. To explain the value of the middle column (tens unit), the researcher said, “If I put one bead here, we have the number ten. If I put another bead here, we have the number twenty.” While saying so, the researcher put two beads in the middle column and said, “Can you please make the number thirty for me?” Then, the test began with the researcher putting the equal number of beads in the ones and tens columns and saying, “Now, I have the number {[N-ty] [N]}.” To ensure that the child remembered the number, the experimenter asked, “What number do I have?” The trial was repeated until the child responded correctly to the memory-check question. Then, the experimenter put one bead in the position of ones or tens columns and asked, “Is the new number {[N-ty] [N + 1]} or {[N + 1-ty] [N]}?” The numbers represented in the abacus were 23, 43, 65, and 45, and numbers in the question were 23/32, 34/43, 56/65, and 45/54, respectively.

After four trials in the tens condition were tested, another four trials in hundreds were given to the children. The experimenter, while putting two beads in the leftmost column said, “If I put one over here, we have the number one hundred. If I put another bead, it is two hundred.” Then, the experimenter asked “Can you please make the number three hundred for me?” After the child gave the correct answer, the researcher retrieved the three beads to empty the abacus. For the first two of the four trials, the researcher put one bead in the leftmost column (hundreds unit) and the equal number the beads in the tens and ones columns and said, “Now, I have the number {[one-hundred] [N-ty] [N]}.” To ensure that the child remembered the number, the experimenter asked, “What number do I have?” The trial was repeated until the child responded correctly to the memory-check question. Then, the experimenter put one bead in the position of ones or tens and asked, “Is the new number {[one hundred] [N-ty] [N + 1]} or {[one hundred] [N + 1-ty] [N]}?” For the last two trials, the researcher put the same number the beads in all three columns and said, “Now, I have the number {[N hundred] [N-ty] [N]}.” To ensure that the child remembered the number, the experimenter asked, “What number do I have?” Then, the experimenter took two beads, putting one bead in the ones column and the other either the tens or the hundreds column, and asked, “Is the new number {[N + 1 hundred] [N-ty] [N + 1]} or {[N hundred] [N + 1-ty] [N + 1]}?” The numbers represented in the abacus were 154, 167, 323 and 566, and the numbers in the questions were 145/154, 167/176, 233/323 and 665/566, respectively. The order of the alternatives in the questions was counterbalanced within and between children.

This task consisted of eight, two-alternative forced-choice trials, and the accuracy that corresponds to the number of correct answers divided by the total number of trials was computed for each child's performance measure.

2.1.2.5. Give-a-Number (Give-N). This task evaluated children's understanding of the counting principles, which was first introduced in Wynn (1990). Here, we used a variant of this task used by Cheung et al. (2017) to test whether or not a child was a cardinality-principle (CP) knower. We used this simpler version because this task only served to ensure that our participants were CP-knowers. The materials used in this task were a set of 12 small, identical plastic bears. On each trial, the researcher asked the child, “Can you give me N?” After the child's response, the experimenter asked, “Is that N? Can you count and make sure?”, motivating the child to count the bears. If the child realized they made an error, the experimenter allowed the child to change the response. Children were tested on four trials and asked to give the following number in that order: 7, 8, 7, and 8. Only those who successfully retrieved the correct number of items in all four trials were considered CP-knowers. Note that this task made explicitly clear that the children count items one by one. Thus, if this task was given prior to the Give-N10 task, it could have influenced children's behavior when the strategy of counting one by one is not optimal in the Give-N10 task. Therefore, this task was administered last.

The data are available in a public repository, 10.17605/OSF.IO/6GDC8.

2.2. Results

2.2.1. Give-a-Number task

All the children correctly gave 7 and 8 objects in the Give-N task. Thus, according to the criteria used in previous studies (Cheung et al., 2017; Davidson et al., 2012), all were considered cardinality principle (CP) knowers.

2.2.2. Unit task

The Unit task measured children's ability to infer the cardinality of the next number within local counting sequences (Cheung et al., 2017; Davidson et al., 2012; Sarnecka & Carey, 2008). The majority of the

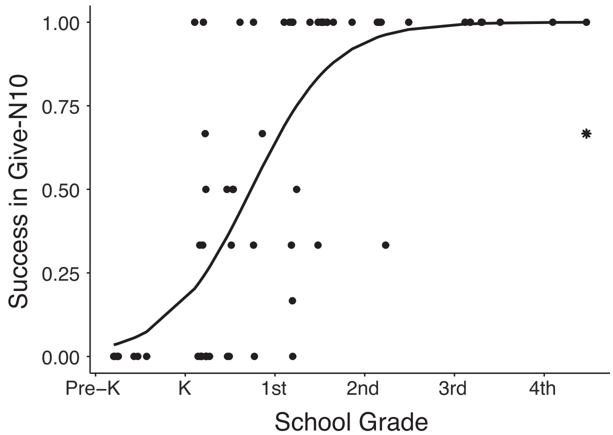

Fig. 2. Success in the Give-N10 task as a function of school grade, with the best-fitting sigmoid function. School grade is adjusted by taking the study date into account—in other words by considering the number of months into each child's current grade.

children performed successfully in this task. Specifically, 52 out of 66 children got at least seven out of eight total trials correct. Considering the binomial distribution $B(8, 0.5)$ with alpha = 0.05, these 52 children were considered to have passed the Unit task. Fourteen children (three preschoolers, ten kindergarteners, and one 1st grader) fell below this criterion. Percentage correct in the Unit task was strongly associated with school grade ($\beta = 0.06$, $t(64) = 4.20$, $p < 0.001$).

2.2.3. Give-a-Number Base-10 (Give-N10) task

This task evaluated children's cardinal knowledge of the syntactic structure of complex numerals. As shown in Fig. 2, approximately half (31 out of 66) of the children received a perfect score in this task, meaning that those children when asked to give 16, 27, and 32 items gave the correct numbers while maximizing the strategy of retrieving the sets of 10 items (see Methods for details). Twenty children failed to use such a strategy completely. The remaining (15) children fell in between the two end points.

Note that, surprisingly, one fourth grader in our sample did not receive a perfect score following our scoring criteria (note the data point in asterisk in Fig. 2). That child did not receive any credit for the 16-item question according to our scoring criteria (but full credits for all other numbers) because instead of retrieving a set of ten items and then retrieving six individual items, she retrieved two groups of three items and then counted ten more items. In fact, that pattern of behavior demonstrates a unique composition of the word *six*[*teen*] as well as an understanding of the decompositionality of numbers (in this case six). We speculate that, while this child did not follow the rather strict scoring criteria we set, she has sophisticated knowledge in numerical syntax. Based on such rationality, we treated this child's performance as a perfect score in subsequent statistical analyses, but it should be noted that the inferential results were identical regardless of whether the child is given a full or partial credit on this task.

In order to characterize the developmental trajectory of performance in the Give-N10 task, we fitted the following sigmoid function assuming that the child transitions from having no knowledge of the syntactic structure of cardinal numbers to having full knowledge of it (minimum and maximum values set to 0 and 1, respectively):

$$f(x) = \frac{1}{1 + e^{-b(x-a)}}.$$

The result indicated the estimated midpoint (a) of this transition at around 8 months into kindergarten with the estimated slope (b) of 2.17. The midpoint in terms of age in our sample was 6 years and 1.6 months.

Underlying our central hypothesis that children's cardinal understanding of numerical syntax is a critical condition for the acquisition of number concepts is our implicit assumption that the performance in the Unit task is not a sufficient measure for assessing children's knowledge about the generative properties of natural numbers. This is because the Unit task requires minimal knowledge about the syntactic structure of complex numerals (see Section 2.3 for more discussion on this). We thus evaluated the relationship between children's performance in the Unit and Give-N10 tasks. If our reasoning that passing the Unit task is not sufficient to understand the syntactic structure of cardinal numbers, then children who pass the Give-N10 task should pass the Unit task and not vice versa. This pattern was indeed evident. Thirty-two out of 33 children who passed the Give-N10 task also passed the Unit task. Of 33 children who failed the Give-N10 task, 20 passed and 13 failed the Unit task. The mid-P McNemar test for the two tasks showed that the marginal proportions were significantly different from each other ($p < 0.001$). These results are consistent with the idea that passing the Unit task is only a necessary but not a sufficient condition for understanding the syntactic structure of cardinal numbers.

There have been previous approaches to evaluate children's understanding of the syntactic regularities in number words (such as rote counting). Give-N10 goes beyond these previous approaches, as this task assesses the *cardinal* understanding of complex numerals by asking children to produce a set of its cardinal value. Nevertheless, given the novelty of the Give-N10 task, it is yet unclear whether the performance in this task is indeed founded on children's knowledge of syntactic regularities in number words. Thus, to better understand what Give-N10 is assessing, we tested two other tasks requiring numerical syntactic knowledge (Interval-Counting and Number-Multiplier-Syntax) and their relation to children's Give-N10 performance.

2.2.4. Interval-Counting task

Observing children's acquisition of the counting sequence is a unique way to glimpse into their understanding of the syntactic regularities (and irregularities) of complex numerals (Gould, 2017). In the Interval-Counting task, we asked children to rote count within three intervals (from 15 to 24, from 37 to 51, and from 95 to 111; see Methods for details) each of which contain some “hurdle” points according to previous work (e.g., Gould, 2017). In the first interval, almost all children reached the upper limit of 24 (62 of 66), although one stopped at 19 and three at 20. In the interval from 37 to 51, 52 out of 66 children successfully counted up to the number 51, while 10 children stopped at the numbers before a new decade (e.g., the numbers 39 or 49), one child stopped at the number 47, one at 40, one at 38, and another child did not start to count by himself (which we treated as missing value). In the interval from 95 to 111, only about half of the

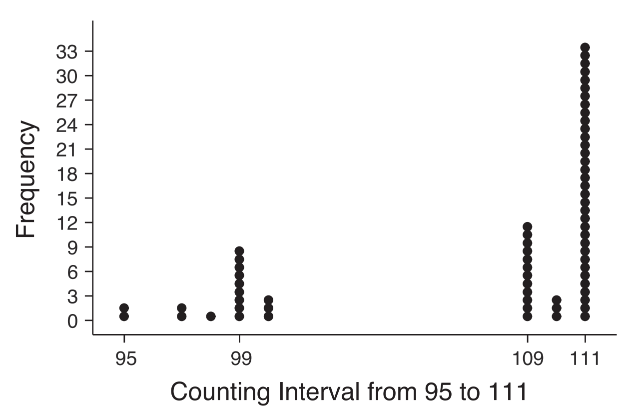

Fig. 3. Highest number reached within the interval from 95 to 111 in the Interval-Counting task. Each data point represents one child. 34 of the 66 children who counted up to 111 were stopped by the experimenter as that was the last number in that interval.

children (34 out of 66) successfully counted up to the number 111. Of those who failed to reach 111, twelve children stopped at the number 109, nine children stopped at 99, three children stopped at the number 100, and five children stopped at the numbers before 99 (see Fig. 3). All the children who were able to count beyond 109 in the third interval successfully counted up to the last number (the number 51) in the second interval, but only 37 of 52 who counted up to the number 51 counted beyond 109 in the second interval. Interestingly, five of those who failed to reach 51 passed the Unit task, correctly solving the questions asking for 57 + 1 or 75 + 1 in the Unit task. This result confirms our intuition that a child's performance in the Unit task is not bound by their count list and suggests that sequential knowledge at the *ones* level is enough to succeed in the Unit task.

The pattern in which children stop at the last number before some of the decades (39, 49, or 109) replicates and confirms previous reports (Fuson, 1988; Gould, 2017; Siegler & Robinson, 1982), and also illustrates children's understanding of the syntactic rule (or lack thereof) for building complex numerals. In subsequent analyses, we used the Decade Transition (DT) score as a summary measure for each child's counting fluency especially concerning the child's knowledge of numerical syntax. Each of the decade transitions in all the intervals (19, 39, 49, 99, 109) was marked 0 (fail) or 1 (pass), resulting in a maximum of 5 points in each child participant (see Methods for more details). The DT score was significantly predicted by school grade ($\beta = 0.63$, $t(64) = 5.10$, $p < 0.001$).

2.2.5. Number-Multiplier Syntax task

This task also evaluated children's understanding of the syntactic regularities in complex numerals. More specifically, it assessed how children associate the sequence of Digits (from one to nine) with Multipliers used in various levels. Children's performance widely varied, but in general was predicted, again not surprisingly, by school grade ($\beta = 0.13$, $t(64) = 5.82$, $p < 0.001$). Thirty-two out of 66 children got at least seven out of eight total trials correct. Considering the binomial distribution $B(8, 0.5)$ with alpha = 0.05, these 32 children were considered to have passed the Number-Multiplier Syntax task, meaning that they are capable of associating numerical sequence correctly with each of the multipliers.

2.2.6. Regression analyses

We then questioned to what extent measures of numerical syntactic knowledge, evaluated in Interval-Counting and Number-Multiplier Syntax, are related to children's understanding about the embedded structure of cardinal numbers, evaluated in Give-N10. Indeed, regression models showed that performance in the Interval-Counting and the Number-Multiplier Syntax tasks predict children's performance in Give-N10. In one model (Table 1, Model 1), children's DT score was a significant predictor of the Give-N10 performance even after controlling for school grade ($\beta_1 = 0.11$, $t(63) = 3.50$, $p < 0.001$). In another independent model (Table 1, Model 2), children's performance in the Number-Multiplier Syntax task was dichotomized (whether or not the child passed the task based on a cumulative binomial distribution) and entered into the model with school grade and the interaction term as a covariate. Whether or not children pass the Number-Multiplier Syntax was a significant predictor of the Give-N10 performance even with other covariates ($\beta_1 = 0.63$, $t(62) = 2.59$, $p = 0.012$).

In the third regression model, performance measures from both Interval-Counting and Number-Multiplier Syntax were entered simultaneously into one model with school grade as a covariate (see Model 3 in Table 1) to predict the Give-N10 performance. The DT score ($\beta_1 = 0.08$, $t(62) = 2.63$, $p = 0.011$) and the dichotomized Number-Multiplier Syntax performance ($\beta_2 = 0.32$, $t(62) = 3.66$, $p < 0.001$) remained significant. These results confirmed that what was measured in Give-N10 is associated with children's understanding of the syntactic structure and the syntactic regularities of complex numerals. Critically, however, note that Give-N10 involves the representation of items in a set (cardinality) while such a representation is not involved in the Interval-Counting or Number-Multiplier Syntax tasks.

Table 1
Regression models with the Give-N10 score as the dependent variable.

	Estimate	S.E.	t	p
Model 1 (adjusted R-square = 0.56)				
β_0(Intercept)	−0.43	0.12	−3.71	0.0004***
β_1(DT)	0.11	0.03	3.50	0.0009***
β_2(School Grade)	0.19	0.04	5.26	0.0000***
Model 2 (adjusted R-square = 0.60)				
β_0(Intercept)	−0.26	0.17	−1.47	0.1452
β_1(Number-Multiplier Syntax)	0.63	0.24	2.59	0.0118*
β_2(School Grade)	0.22	0.07	3.13	0.0026**
β_3(School Grade) × (Number-Multiplier Syntax)	−0.09	0.08	−1.08	0.2861
Model 3 (adjusted R-square = 0.63)				
β_0(Intercept)	−0.28	0.12	−2.39	0.0198*
β_1(DT)	0.08	0.03	2.63	0.0107*
β_2(Number-Multiplier Syntax)	0.32	0.09	3.66	0.0005***
β_3(School Grade)	0.12	0.04	3.37	0.0013**

* $p < 0.05$.
** $p < 0.01$.
*** $p < 0.001$.

2.2.7. Decade transitions and performance in Give-N10 task

The results so far indicate that the DT score is a significant predictor of children's Give-N10 performance. However, the DT score is a summarized measure of children's knowledge about the regularities in the counting sequence, so it is unclear which kind of syntactic rules for forming complex numerals give rise to children's understanding of the embedded structure of cardinal numbers. In fact, it is conceivable that success in some particular decade transition may be more informative for predicting the Give-N10 performance.

To address this question, we performed a post hoc analysis examining the association between whether or not a child passed Give-N10 (0.66 or above) and whether or not the child successfully produced the decade name in each of the decade transitions in the Interval-Counting task in a cross tabulation. The chi-square values of these associations are plotted in Fig. 4. The highest association was found between counting past one hundred nine and passing Give N10 ($\chi_{(1)}^2 = 19.93$, $p < 0.001$). The other transitions (counting past 39, 49 and 99) resulted in lower chi-square values $\chi_{(1)}^2$ (7.23, 10.01, and

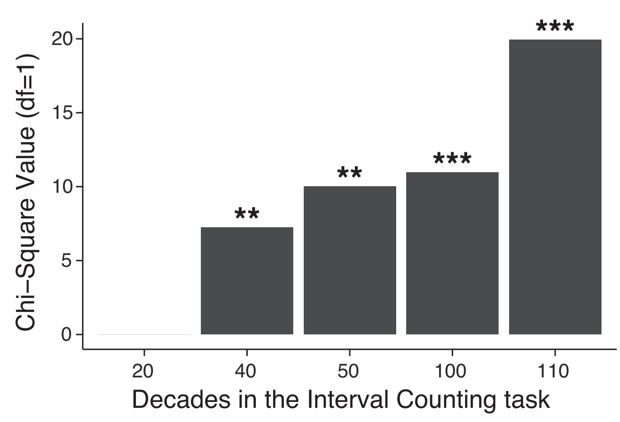

Fig. 4. Association between successful decade transition and Give-N10 performance in Study 1. For each decade transition, a chi-square test for independence was performed by cross-tabulating two dichotomous variables: pass/fail decade transition and pass/fail Give-N10. The greatest association was found at the decade 110, which indicates that children who can count beyond 109 reliably solved the Give-N10 task.

10.97 respectively). These results suggest that counting beyond one hundred nine is a unique developmental milestone. We return to this point in Discussion.

2.3. Discussion

Despite the theoretical importance of, and scholarly interest in, the successor principle, little empirical work has directly investigated this logical foundation of natural numbers. To our knowledge, only two types of empirical approaches have directly investigated the acquisition of the successor principle. The first type is via the Unit task, first introduced by Sarnecka and Carey (2008) and used in other following studies (Cheung et al., 2017; Davidson et al., 2012), including the current study. The structure of this task mimics how the successor principle is defined (i.e., making a semantic induction from N to N + 1) and thus is claimed to measure children's knowledge of the successor principle (Le Corre & Carey, 2007, 2008; Sarnecka & Carey, 2008). However, this task by design does not require children's ability to make a semantic induction over all possible numbers, nor over the children's own count list (Cheung et al., 2017; Davidson et al., 2012). This is because knowing how to make an induction over a *local* sequence of numbers from one to nine is sufficient to solve this task successfully, as long as the child has some knowledge about the structure of complex numerals. Take the example illustrated in Fig. 1A. To fully understand the recursive structure of cardinal numbers, a child must understand that the same lexical category Digit used in ones is used to build tens. Such knowledge about the recursive structure is not needed to pass the Unit task but is needed to pass the Give-N10 task (and pass some critical decade transitions reliably). In other words, a child can solve the Unit task with a minimal understanding that, say, the numeral twenty-five contains the word five (along with some other word) and that an increment of one item in the environment is associated with the next number (i.e., six) in the local numerical sequence. In fact, our results show that some children who pass the Unit task (with numbers as high as 75) remain unable to count fluently. Two of those children stopped at 39, two children stopped at 49, and one child stopped at 40 in the Interval-Counting task, suggesting that the Unit task can be solved in intervals where the counting sequence is not yet mastered. These results make it difficult to argue that the Unit task measures any abstract knowledge about the successor principle nor the recursive operations of numerals. Moreover, this task cannot explain how children come to understand that any number has a successor.

The second area of work targets children's declarative knowledge of infinity, exploiting the idea that one of the end states of the acquisition of the successor principle is the comprehension of infinity. Cheung et al. (2017), for instance, inquired children's knowledge about infinity by asking them about the biggest number they know and about adding one to that number. Falk (2010) also devised a similar approach to assess whether, and when, children grasp the concept of infinity. While these studies have provided meaningful results, the interpretation requires some caution. In some items of the infinity task, the experimenter explicitly asks the children about an arithmetic operation (i.e., adding one to a number). Even if the children answered this type of question correctly and they understand that adding one to a big number generates a higher number, there is a difference between knowing the successor rule and knowing the result of an arithmetic operation. In addition, the introspective nature of this task restricts the interpretation about children's precise knowledge level. Most importantly, even if this approach may allow us to identify the final state of the acquisition of number concepts it is limited in explaining how that state is achieved.

As described above, the prevalent assumption in the field has been that children's concept of nature numbers comes from the understanding of the successor principle or, in an empirical sense, from understanding the next number (see also Barner, 2017; Carey & Barner, 2019; Chomsky, 2005; Rips, Bloomfield, & Asmuth, 2008). However, tasks that inquire children about the next number (e.g., the Unit task) fail to show that the knowledge of the successor is directly related to children's understanding of natural numbers. Such results invite us to rethink the appropriateness of the successor principle for explaining children's acquisition of number concepts. The idea of the successor principle in its abstract form, as described in the Peano's axioms, is a formal mathematical definition that has little to do with the structure of the symbolic system used to learn and mentally represent numbers, which has also been argued in recent philosophical analyses of the topic (Buijsman, 2018, 2019a, 2019b). Thus, the successor principle may not be the right framework to investigate the mental representation of number. Instead, we argue that a more plausible candidate for the mechanism underlying number concept acquisition is the comprehension of the syntactic structure of cardinal numbers. After all, the analysis of recursive structure of numerals is only possible after the analysis of its syntax.

Our study is designed to assess children's understanding of numerical syntax as an important step towards understanding the state of their number concepts. In particular, we devised and ran a task (Give-N10) that evaluates children's knowledge of the base-10 number system. Specifically, to succeed in this task, children must parse a complex numeral and understand the cardinal meaning of each element in that numeral. Our results demonstrate that relatively older children, first graders and above in our sample, are able to maximally perform this task using the base-10 number knowledge. It is worth reiterating that virtually everyone who passed this Give-N10 task passed the Unit task, but the reverse was not true. This pattern again speaks to the idea that the Unit task is not sufficient to demonstrate children's number concepts.

Considering that substantial formal math education begins in kindergarten, it is not surprising to see that the performance in Give-N10 soars rapidly in kindergarten and first grade (Fig. 2). What is important, however, is that the performance in this task is highly predicted by children's DT score and Number-Multiplier Syntax performance even when school grade was controlled for (Table 1). These results indicate that even after removing the effect of schooling, the comprehension of numerical syntax and its associated cardinality is founded on the understanding of the syntactic regularities in the counting sequence. Thus, the results further demonstrate that counting is not a mere recitation of a memorized sequence. Rather, it is a demonstration of children's knowledge about the syntactic regularities of numerals.

One interesting pattern about children's counting fluency is worth noting. As in previous reports (e.g., Gould, 2017; Siegler & Robinson, 1982), many of our children stopped at the number one hundred nine. More unique to our results is that counting beyond one hundred nine was a critical determiner for passing Give-N10. Gould (2017) suggests that children's knowledge of English grammar for tens inhibits the production of "ten" in "one hundred ten." In tens, children use the sequence between one and nine (e.g., from thirty-one to thirty-nine) but the use of "ten" after the Digit "nine" is not allowed. Indeed, when children produce an ungrammatical cardinal number such as "thirty-ten," they are often corrected. According to this idea, this hesitation to say "one hundred ten" implies that those children do not have a correct cardinal understanding of the word "one hundred." One possibility is that they may be treating it like "ten-ty" following "nine-ty," which is especially possible if children do not have a firm understanding that the same one-through-nine sequence is recursively used in the ones level and in the decades level (and beyond of course). In addition, many in English numerals below one hundred have idiosyncratic, non-analytical properties that appear to obscure or block the child's capacity to realize the system which is systematic when several levels of embedding are entailed, as with numbers that embed tens, hundreds, and even thousands within a single number.

The results so far suggest that children's understanding of the embedded structure of cardinal numbers can be measured empirically using Give-N10, which seems to be founded on their knowledge about the syntactic regularities of complex numerals. In our sample of

English-speaking children, it was not until the age of 6 years on average (several months into kindergarten) that the majority of the children passed the Give-N10 task. These findings suggest that a syntactic understanding of the meaning of number words (even within the ranges of children's count list) comes much later in development than previously thought, and that understanding the embedded structure of numbers may be a critical cornerstone for the acquisition of number concepts. However, as novel as the task was, we conducted a second independent study to test whether Give-N10 provides meaningful, interpretable results in another sample of children using another language.

3. Study 2

A follow-up replication study was conducted to test whether Give-N10 provides a meaningful and interpretable measure of children's understanding of the embedded structure of cardinal numbers. In particular, we focused on capturing the Give-N10 performance in Korean-speaking children around the age of five and six and tested its association with their counting ability measured by the decade transition score.[6] We ran this replication study in Korea as it was a unique opportunity to exploit the regularity of the Korean number system.

The Korean language has two sets of numerals, both of which used regularly in day-to-day life but in slightly different circumstances. The native Korean system contains lexically unique decade names which are non-transparent like eleven and twelve in English cardinal numbers, and the Sino-Korean system (derived from Chinese) contains decade names that are compositional based on the base-10 structure. The Sino-Korean numerals are regular in both teens and decades. Teens are a combination of the word for ten (i.e., sib) and a word in the ones class (e.g., seventeen is ten-seven which is sib-chil in Korean). Decades are a combination of a word in the ones class and the word for ten (e.g., seventy is seven-ten or chil-sib). In other words, it is easy to see the arithmetic operations that make up the base-10 number system in Korean.

By evaluating children's performance in Give-N10 and its association with their counting fluency in Korean-speaking children, we aimed not only to validate the original results but also to better understand the influence of numeral system regularity on children's acquisition of the embedded structure of cardinal numbers.

3.1. Methods

3.1.1. Participants

Thirty-one kindergarteners (19 boys and 12 girls; mean age = 5 years 11 months; age range = 5;3–6;5) participated in this study. Children were recruited for the study from three different public kindergartens in Seoul, South Korea with parental consent, and it was conducted in the schools. In South Korea, the academic calendar runs from March to December. Children who turn seven in the upcoming calendar year (from Jan to Dec) enter first grade in elementary school, while younger children are eligible to enroll in public kindergartens. We aimed to recruit and run kindergarteners because the age range of these children in Korea best matched the age range in which we observed the widest variability in English-speaking children's Give-N10 performance (see Fig. 2). Children completed three tasks in the following order: Interval-Counting, Give-N10, and Give-N. All procedures were approved by the University of Massachusetts Institutional Review Board. The three tasks were run identically to that of Study 1, with minor differences. In the following section, we limit our description of the procedure and tasks to those differences. Some of the children were given another task after Give-N10, and some 5-year-olds were given yet another task prior to Give-N10. These tasks are not discussed here as they are outside the scope of the current paper, but we note that none of these other tasks involved counting objects.

3.1.2. Procedure and tasks

3.1.2.1. Interval-Counting. Children were asked to count the following intervals in that order using the Sino-Korean system: from 1 to 11, from 16 to 32, from 37 to 51, from 95 to 111, and from 285 to 311. Children were asked to count all the intervals regardless of whether or not they made errors in any of the intervals. Unlike in the case of the first interval (1–11) where the first three numbers were introduced by the experimenter, only the very first number was introduced in the rest of the intervals. When the child counted all the numbers in a given interval without an error, the child proceeded to the next interval. When the child made an error in a given interval, the highest number (N) that the child counted up to before the error was first recorded. In that case, we asked “What number comes after N?” without hinting at the next number after N. If the child did not know the next number, we stopped the interval and proceeded to the next interval. If the child knew the next number and resumed counting the numbers, we let the child continue until the second error, if there were any. We urged the child similarly at the second error. At the third error, if it existed, we stopped the interval and proceeded to the next interval by saying along the lines of “Okay, then let's now count from [ten-six] (which is a literal translation of the word sixteen in Korean).” As in Study 1, the decade transition (DT) scores were quantified as a measure of children's understanding of the regularities of the numerical syntax. The points of decade transition were 9, 19, 29, 39, 49, 99, 109, 289, 299, and 309. Thus, the highest score that a child can reach was 10.

3.1.2.2. Give-a-Number Base-10 (Give-N10). The procedure for Give-N10 was identical to that of Study 1, except a different set of cardinal numbers were used. Instead of using four distinct groups of ten squares, Study 2 incorporated five distinct groups of ten squares, and asked children to give 27, 32, and 45 squares. Specifically, the experimenter first modeled the case of retrieving 31 squares, and children were asked to give 1, 7, and 10 item(s) so that they had an opportunity to retrieve individual items from a group of ten, as well as retrieve ten items at once (for those who can). Then, as the experimental trials, the experimenter asked the children to move 27, 32, or 45 squares to a designated spot.

We used the number 45 instead of the teen number used in Study 1 for two reasons. First, native Korean numerals are typically preferred when it comes to counting items below twenty objects. Because we used Sino-Korean numerals, we avoided the numbers below twenty. Second, we reasoned that Korean-speaking children would have a better understanding of the numerical syntax in Sino-Korean numerals owing to the regularity of its numerals. Thus, we reasoned that Korean-speaking children would be able to manage an even higher number like 45 compared to the highest number 32 used in Study 1. The coding scheme for the performance in this task was identical to that in Study 1.

3.1.2.3. Give-a-Number (Give-N). The procedure of the Give-N task was identical to Study 1. While counting objects within a small range of numbers (1–20 objects) is most natural using the native Korean numerals, we used the Sino-Korean numerals to be consistent with the rest of the tasks. While this may have sounded unconventional, it should not have interfered with our primary purpose, which is to assess children's knowledge about the counting principles.

3.2. Results

3.2.1. Give-a-Number task

All but three children successfully gave 7 and 8 objects and were categorized as CP-knowers.

[6] As we ran the study in local kindergartens, our time with each child was limited. Thus, we chose Interval Counting over Number Multiplier Syntax for its simplicity and dropped the Unit task as the results concerning that task in Study 1 was already extremely robust (see the mid-P McNemar test results in Section 2.2.3).

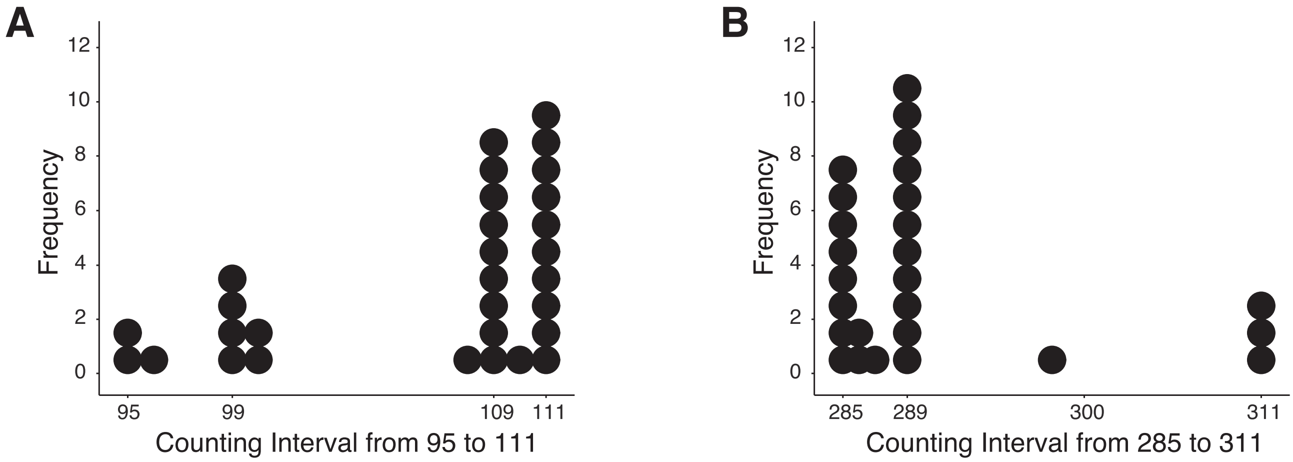

Fig. 5. Highest number reached within the interval from 95 to 111 and from 285 to 311 in the Interval-Counting task in Study 2. Each data point represents one child. Children were stopped at the numbers 111 and 311 in these two intervals.

3.2.2. *Give-a-Number Base-10 (Give-N10) task*

Children's Give-N10 performance expectedly ranged across the scale, as the age range of the Korean kindergarteners tested was narrow (only 5- and 6-year-olds). Note that this age range resulted in the most variability in the Give-N10 performance in Study 1. Seven out of 31 children failed to use the base-10 structure in all three trials, ten children succeeded in all the trials, and the rest fell in between the two end points. We then addressed the more interesting question of whether the individual differences in this Give-N10 performance can be explained by children's counting ability as in Study 1.

3.2.3. *Interval Counting task*

We first validated the stopping points in the Interval-Counting task that were observed in Study 1. In the first interval (1–11), all but one child counted up to 11. In the second interval (16–32), 8 out of 31 children did not reach the upper limit, half of them stopping at numbers ending with 8 or 9. In the third interval (37–51), 6 out of 31 children did not reach the upper limit, with most of them stopping at 39. The highest number reached within the fourth (95–111) and fifth (285–311) intervals are plotted in Fig. 5. Across both intervals, the prominent stopping points were 99, 109, and 289, consistent with the idea that children find decade transitions most difficult when rote counting.

Children's counting fluency was quantified using the Decade Transition (DT) score. The average DT score across all children was 5.6. If we are to assume the case that children who pass the higher DT always pass the lower DT, the DT score of 5.6 can be interpreted as having a highest count somewhere between 49 and 99. More importantly, to follow-up our results in Study 1, we tested whether DT score predicted children's Give-N10 performance; it did indeed. DT score was a significant predictor of Give-N10 both with ($\beta = 0.26$, $t(28) = 3.10$, $p = 0.004$) and without ($\beta = 0.29$, $t(28) = 3.66$, $p < 0.001$) using age as a covariate.

As in Study 1, we examined the relationship between children's Give-N10 performance and successful decade transition in each of the decades in the Interval-Counting task. Fig. 6 illustrates that the only association was found between counting past ninety nine and passing Give N10 (${\chi_{(1)}}^2 = 4.62$, $p = 0.031$).

It should be noted that the highest association between successful decade transition and Give-N10 was present in the transition at one hundred nine in our English-speaking children in Study 1 (Fig. 4), while Korean children showed that highest association in the transition at one hundred (Fig. 6). These results suggest that Korean-speaking children understand the embedded structure of cardinal numbers earlier in the acquisition path than their English-speaking peers. We sought to reason this on a statistical basis by testing whether the chi-square value

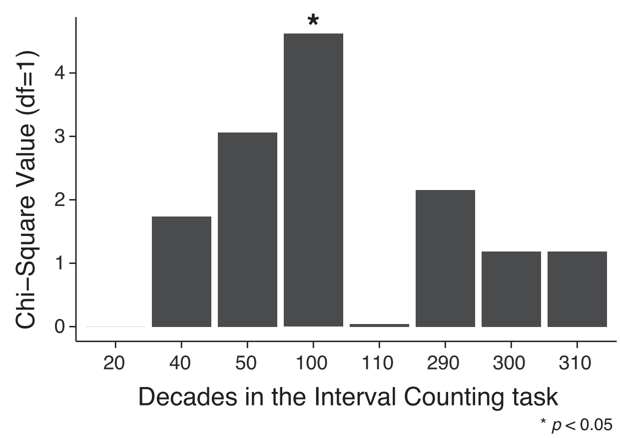

Fig. 6. Association between successful decade transition and Give-N10 performance in Study 2. For each decade transition, a chi-square test for independence was performed by cross-tabulating two dichotomous variables: pass/fail decade transition and pass/fail Give-N10. The only association was found at the decade 100, which indicates that children who can count beyond 99 reliably solved the Give-N10 task.

difference between the decades 100 and 110 was indeed different between the two languages, using a non-parametric resampling method. In each language, the difference between two chi-square values was taken: the one measuring the association between successful transition in 110 and Give-N10 (χ^2_{110}) minus the other measuring the association between successful transition in 100 and Give-N10 (χ^2_{100}). Then, the θ statistic was defined as the difference of the differences, $[\chi^2_{110} - \chi^2_{100}]_{\text{English}} - [\chi^2_{110} - \chi^2_{100}]_{\text{Korean}}$, which effectively quantifies the interaction between language and decade. We then tested the effect of θ under the null hypothesis that the difference between the two chi-square values is the same between the two languages. Thus, the null distribution was generated by relabeling the language variable 30,000 times. The observed θ was 13.535, and the probability of the simulated θ under the null hypothesis exceeding the observed θ was $p = 0.0091$. These results suggest that language indeed leads into a different pattern of association between Give-N10 and decade transitions at 100 and 110.

3.3. *Discussion*

As in Study 1, Give-N10 performance in Korean children was predicted by their understanding of the syntactic regularities in the counting sequence measured by the decade transition score in rote counting. This positive relationship found in speakers of the two languages bolsters the idea that Give-N10 is indeed a valid way of assessing children's understanding of the embedded structure of cardinal numbers and that this understanding comes from the recursive structure of the counting sequence.

At the same time, results from Study 2 provide some evidence that the regularity of a language's number system facilitates children's cardinal understanding of numerical syntax. Indeed, previous studies have demonstrated that the regularity of the numerals influences children's rote counting ability. For example, Korean-speaking preschoolers outperform English-speaking preschoolers in counting (Miller & Stigler, 1987) and solving single digit arithmetic problems (Fuson & Kwon, 1992). In another previous study, Miura, Kim, Chang, and Okamoto (1988) compared the preference of child speakers of English and Asian languages using a set of tens and units when asked to show double digit numbers such as 11, 13, 28, 30, and 42. Asian children preferred to compose the numbers with both tens and units blocks, whereas English speaking children preferred to compose the numbers primarily with units blocks.

Our current results suggest that the point at which children understand the embedded structure of cardinal numbers may differ slightly depending on the regularity of the numerical syntax in their language. In English-speaking children (Study 1), the most prominent point in the Interval Counting task was one hundred nine. Those who were able to count beyond one hundred nine were most likely to succeed in the Give-N10 task. In Korean-speaking children (Study 2), it was the number ninety nine that best distinguished children who passed or failed the Give-N10 task. This pattern supports the idea that the regularity of the numerical syntax facilitates children's learning of the recursive and embedded structure of cardinal numbers. For instance, because it is so apparent in Sino-Korean numerals that the same lexical element from one to nine is used to express the decade numbers, it may be more straightforward for a child to think that the numbers coming after ninety nine (goo-sib-goo)—after being able to count past chil-sib (seven-ten), pal-sib (eight-ten), and goo-sib (nine-ten)—must be reaching another level of numerical expression, thus eventually understanding the recursive structure. In addition, the Sino-Korean cardinal numbers make use of the multiplier for 10 (i.e., sib) both as a bare multiplier in the range from 11 to 19 (e.g., $[_{\text{Number}}\ [_{\text{Phrase}}\ [_{\text{M}}\ \text{sib}][_{\text{Number}}\ [_{\text{Digit}}\ \text{ee}]]]$ for the number 12) and as a multiplier in the range from 20 to 99 (e.g., $[_{\text{Number}}\ [_{\text{Phrase}}\ [_{\text{Number}}\ [_{\text{Digit}}\ \text{ee}]][_{\text{M}}\ \text{sib}]]]$ for the number 20). The consistent use of the same word in two different structures may facilitate children's understanding about the idea that the lexical elements expressing cardinal numbers are combinatorial.

4. General discussion

We evaluated children's cardinal understanding of the syntactic structure of complex numerals using a novel task, Give-a-Number Base-10, in two independent studies conducted in two other languages (English and Korean). This task goes beyond children's ability to infer the next number in a local counting sequence (assessed by the Unit task). This task also goes beyond children's mere knowledge about numerical syntax (assessed by the Interval Counting and the Number Multiplier Syntax tasks), as it involves their understanding of cardinality. However, the performance in this task is highly associated with children's understanding of syntactic regularities in complex numerals in both languages.

Give-N10 was designed to assess children's understanding of, or sensitivity to, the arithmetic operations of the base-10 number system. It should be noted that there is a vast amount of literature on children's understanding of the base-10 knowledge using place value in multi-digit Arabic numerals (Fuson, 1990; Kamii, 1986; Miura & Okamoto, 1989; Ross, 1989). While both the place value systems and the cardinal numbers are external representations of the formal numerical concept, the former is fundamentally different from the latter. For instance, although related, seeing 109 is entirely different from saying or hearing one hundred nine. More precisely, in place value system, space (or place) is central to expressing the power of the base, whereas in cardinal numbers, the power is represented by lexical elements (i.e., Multipliers). Moreover, the structure of the place value systems does not incorporate a sequence, whereas numerical sequence is a defining characteristic of cardinal numbers. Unlike what is studied about children's learning the place value system, relatively little is known about how children acquire the base number knowledge from cardinal numbers. Our work paves the way for this venture.

In a way, our approach and conceptualization of children's knowledge about the embedded structure of cardinal numbers is similar to how Fuson et al. (1997) have theorized children's development of understanding multi-digit numbers. In particular, Fuson et al. (1997) claims that children go through a "unitary" understanding of multi-digit numbers to a "decade and ones" phase, and eventually to a phase where the tens and the ones are represented independently ("sequence tens and ones" or "separate tens and ones" phases). In that framework, the state of passing the Give-N10 task is likely to be categorized by this very last stage. Nevertheless, Fuson and colleague's model has been proposed to explain the multi-digit numeral comprehension using three external representations, Arabic numerals, numbers words, and blocks. This model provides little basis for how conceptual representations of complex numerals develop. Our work contributes to this literature by suggesting the importance of numerical syntax for a comprehensive understanding of cardinal numbers. More importantly, we propose that such an understanding is an important basis of children's understanding of generative number concepts.

How is the cardinal understanding of numerical syntax related to the acquisition of number concepts? One possibility is that the recursive properties of syntactic elements of complex numerals could directly give rise to the understanding of the successor principle (e.g., as in the Peano's axioms) as previously implied (e.g., Rips et al., 2008; Barner, 2017). However, this scenario requires a precise explanation for how a recursive hierarchical numeral structure results in an abstract principle. Relaford-Doyle and Núñez (2018) criticize the idea of successor principle as a cognitive mechanism. They provide evidence that mathematical functions for the successor principle of natural numbers are learned only by formal training and that most adults without formal training never develop this abstract function. According to this view, it is unlikely that young children come to understand the abstract nature of the successor principle.

Instead, a plausible explanation for children's acquisition of number concepts concerns how they understand additive and multiplicative compositions of complex numerals that represent cardinal numbers (Hurford, 2007). For example, in order to understand the cardinal number seventy-two, a child must understand that seventy is composed of seven (Number) of the tens (−ty; Multiplier) and that seventy two is composed of the addition of seventy (Phrase) and two (Number). There are several potential developmental stages before comprehension of the relationship with the cardinal number and the arithmetic algorithm of the base-10 number system. Young children at first may not understand that the elements in a complex numeral resemble a hierarchical construction. These children would produce numbers, for example in the current Give-N10 task, by counting one by one until the target number is reached. Children then may come to understand that the constituents in complex numerals represent different categories without fully knowing the arithmetic operations the numerals resemble. These children would be capable of using the counting procedure as an iterative operation in different units. For example, they would count by tens (e.g., 10, 20, 30, …) to reach the target decade and by ones (e.g., 1, 2, 3, …) to reach the final number. Finally, children may come to understand the arithmetic operations that complex numerals represent. In younger children, however, it may be possible that the category Phrase is processed as a repeated addition of tens (for seven times to indicate seventy). As Carraher, Carraher, and Schliemann (1985) showed, repeated addition typically replaces multiplication in everyday mathematics. After this stage, children may come to understand the multiplicative operations represented by the Merge of the Number and Multiplier categories (e.g., [three-ty]). Importantly, our crosslinguistic data suggest that the regularity of numerals plays a critical role in children's learning of these additive and multiplicative complex numeral compositions.

Our data from the Give-N10 task currently cannot distinguish between a completely multiplicative account and a repeated addition account. Nevertheless, the critical element for acquiring number concepts, as we propose, is understanding the meaning (i.e., cardinal value) of each of the recursively embedded number words in a complex numeral and their operational relations. It is worth noting that Spelke (2017) develops a similar hypothesis, proposing that the grammar of conjunctive noun phrases (e.g., two cats and three dogs) and prepositional phrases (e.g., three piles of two waffles) facilitates the acquisition of new cardinal numbers. However, there is no empirical support behind the idea that children learn, for example, the number five from the expression “two and three” or the number six from the expression “three of two.” Our proposal, on the other hand, falls back on the base number system, in that the cognitive processes for multiplicative (or repeated additive) and additive compositions arise from the syntactic structure of complex numerals. Additionally, it remains possible that the development of complex syntax elsewhere in language can facilitate the complex syntax behind numerals.

5. Conclusion

We conclude by proposing a theoretical framework for children's acquisition of number concepts, in particular the generative properties of natural numbers. The core research question at stake is how children understand, in a philosophical sense, the infinite use of finite means. The most dominant idea in the literature is that the successor principle drives the acquisition of number concepts (Barner, 2017; Carey & Barner, 2019; Chomsky, 2005; Rips et al., 2008). However, our theoretical analysis and empirical observations lead us to challenge that view.

First, it is important to note that number is different from numeral, or the external representation of number. The successor principle provides a theoretical explanation for natural numbers, and it has the capacity to generate any natural number regardless of its external representation. In reality, however, children acquire the meaning of numbers through the exposure to and use of numerals instead of having direct access to the abstract representation of number. In addition, as previously mentioned, there is empirical evidence to suggest that even adults may not have a genuine understanding of the successor rule unless they receive a formal training in mathematical induction (Relaford-Doyle & Núñez, 2018). Observations from cross-linguistic studies also speak against the idea of the successor principle. Regardless of their external representative form, if the successor principle is the core mental mechanism for representing natural numbers, it must have been invented/discovered in all languages that have numerals. Consequently, these languages must produce numerals that exhibit the successive properties of natural numbers. However, this is not true, as many languages exist with a finite set of number words (Beller & Bender, 2008; Corbett, 2000; Saxe, 1981). This could mean that these populations have the mental capacity for the successor principle, but their languages do not provide a rule to express large numbers. If so, the successor principle is not sufficient for representing natural numbers. In contrast, it could mean that these populations do not have the mental mechanism for the successor principle because their external representations are bounded. In that case, their concept of numbers is bounded within the external representations, again making the successor principle unnecessary for explaining their number concepts.

All mental knowledge requires mental representation. If we can comprehend an overt representation of a phenomenon, then most likely the overt representation must have a definable relation to that mental representation. Thus, when it comes to the acquisition of number concepts, it is reasonable to posit that the acquisition is directly based on the structure of the external representations of number. In the current work, we have demonstrated that children's cardinal understanding of this embedded structure of numbers—or numerical syntax—can be empirically measured by the Give-a-Number Base-10 task and that this understanding develops prominently in kindergarten-age children. Moreover, this understanding is tightly associated with children's understanding of syntactic regularities in complex numerals, and this association is influenced by the regularity of a language's numeral system. Based on these observations, we propose that children's comprehension and production of the recursive syntactic rules that represent arithmetic operations in a base system is the foundation to their understanding of the generative properties of natural numbers. Further, we propose that the cardinal representation of numerical syntax *is* the structure of numerical thinking at a young age.

CRediT authorship contribution statement

Diego Guerrero: Conceptualization, Methodology, Investigation, Formal analysis, Writing - original draft, Writing - review & editing, Visualization. **Jihyun Hwang:** Methodology, Investigation, Formal analysis, Writing - original draft, Writing - review & editing, Visualization. **Brynn Boutin:** Methodology, Investigation. **Tom Roeper:** Writing - review & editing, Funding acquisition. **Joonkoo Park:** Conceptualization, Methodology, Writing - original draft, Writing - review & editing, Supervision, Project administration, Funding acquisition.

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