Nordic 4 – GR as QFT/EFT   DONOGHUE

Note Title   1/5/2019

Final topics:

Unitarity Methods

Double Copy

Light Bending

Classical physics from loops

Background Field Method and 't Hooft Veltman

~~Ghosts~~

Non-~~local~~ Actions

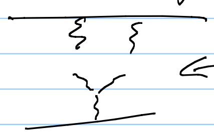
Limits to the EFT

Recall

Tree Theorems

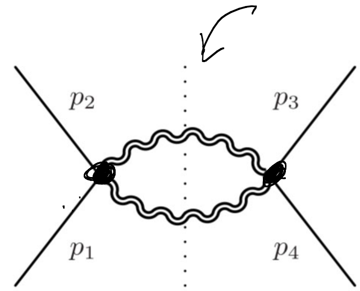
This made me investigate the entire subject in great detail to find out what the trouble is. I discovered in the process two things. First, I discovered a number of theorems, which as far as I know are new, which relate closed loop diagrams and diagrams without closed loop diagrams (I shall call the latter diagrams "trees"). The unitarity relation which I have just been describing, is one connection between a closed loop diagram and a tree; but I found a whole lot of other ones, and this gives me more tests on my machinery. So let me just tell you a little bit about this theorem, which gives other rules. It is rather interesting. As a matter of fact, I proved that if you have a diagram with rings in it there are enough theorems altogether, so that you can express any diagram with circuits completely in terms of diagrams with trees and with all momenta for tree diagrams in physically attainable regions and on the mass shell. The demonstration is remarkably easy. There are several ways of demonstrating it; I'll only chose one. Things propagate from one place to another, as I said, with

Unitarity Method



$\Leftarrow$ on shell

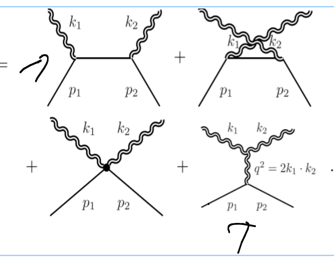
no ghosts!



1) Dispersion relation

$$\mathrm{Amp}(q^2) = \frac{1}{\pi}\int_0^\infty dt \, \frac{\mathrm{Im}\, \mathrm{Amp}(t)}{t - q^2 + \epsilon}$$

$iM^{(s)}_{grav.}(p_1, p_2, k_1, k_2) =$



amplitudes

P.V. $\Rightarrow c_B I_2 + c_T I_3 + c_{Box} I_4$

— recognize by cuts

$$iM_0^{\text{tree}}(p_1, p_2, k_1^+, k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1 \cdot k_2)}\frac{m^4\,[k_1\,k_2]^4}{(k_1 \cdot p_1)(k_1 \cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1, p_2, k_1^-, k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1 \cdot k_2)}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1 \cdot p_1)(k_1 \cdot p_2)},$$

Spinor helicity

$$\Longrightarrow \frac{41}{10\pi}\,\frac{G\hbar}{r^2} \quad \checkmark$$

Universal soft theorems at 1 loop

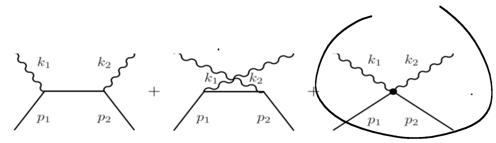
$\rightarrow$ Soft theorems — Low, Jackiw, Gross, Weinberg '60s

$\Longrightarrow$ loop also universal — Holstein Ross $\checkmark$

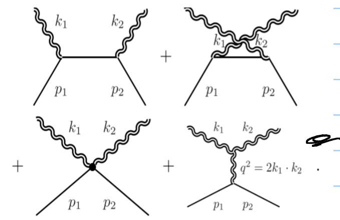
Double copy

E+M

$i\mathcal{M}^{(s)}_{EM}(p_1,p_2,k_1,k_2) =$



$i\mathcal{M}^{(s)}_{grav.}(p_1,p_2,k_1,k_2) =$

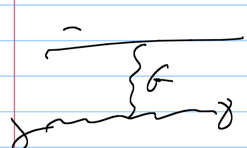


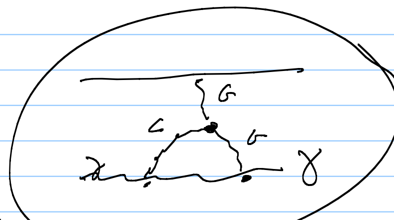
$$M^{(s)}_{grav} = \frac{\kappa^2}{8e^2}\,\frac{p\cdot k_1\, p\cdot k_2}{k_1\cdot k}\,M^{(0)}_{EM}\; M^{(s)}_{E}$$

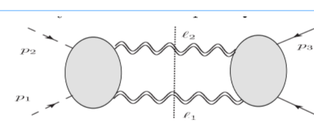
$$\epsilon_{\mu\nu}(+2) = \epsilon_\mu(+1)\,\epsilon_\nu(+1)$$

Bending of Light

- amplitude on shell
- mult.







$$i\mathcal{M}^{[h^+(k_1)h^-(k_2)]}_{[\gamma^+(p_1)\gamma^-(p_2)]} = \frac{\kappa^2}{4}\,\frac{[p_1\,k_1]^2\,\langle p_2\,k_2\rangle^2\,\langle k_2|p_1|k_1]^2}{(p_1\cdot p_2)(p_1\cdot k_1)(p_1\cdot k_2)},$$

Not universal

Not geodesic motion

classical

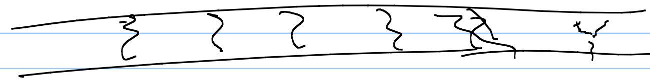
$$b_u^0 = 3/40$$

$$b_u^\gamma = -11/120$$

tree

$$\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2 \times\Big[\frac{\kappa^2}{t} - \kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}} - \hbar\kappa^4\frac{15}{512\pi^2}\log\left(\frac{-t}{M^2}\right) + \hbar\kappa^4\frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right) - \hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right) - \kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\Big]$$

Eikonal Method



$$\mathcal{M}(\boldsymbol{b}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \mathcal{M}^{(1)}_{sum}(\boldsymbol{q})$$

$$\mathcal{M}(\boldsymbol{b}) = 2(s - M^2)\left(e^{i\chi_1(\boldsymbol{b})} - 1\right)$$

phase

stationary phase –

classical NLO classical quantum

$$\theta \simeq \frac{4G_N M}{b} + \frac{15}{4}\frac{G_N^2 M^2 \pi}{b^2} + \left(8bu^S + 9 - 48\log\frac{b}{2b_0}\right)\frac{\hbar G_N^2 M}{\pi b^3} + \dots$$

IR

Classical Physics from Quantum Loops

Folk theorem: Loop expansion = $\hbar$ expansion

— not true

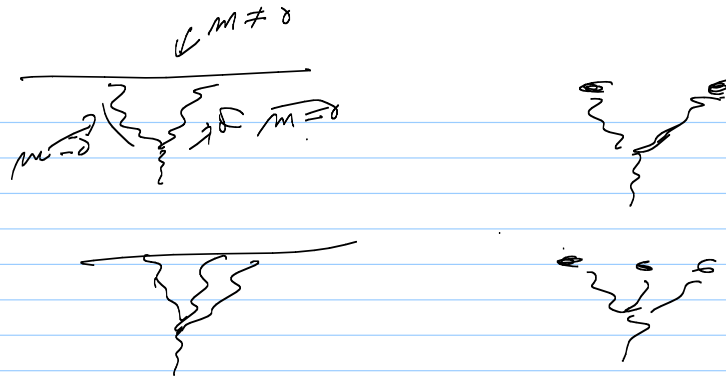
Iwasaki, Gupta + Radford → classical part

+ Modern

What goes wrong?

$$\hbar\, \bar{\psi}\left(i\not{\partial} - \frac{m}{\hbar}\right)\psi$$

At one loop $\hbar \sqrt{\frac{m^2}{\hbar^2 q^2}} \sim \frac{m}{\sqrt{q^2}}$



static sources in classical theory

Background Field Method

't Hooft Veltman

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(x) + \kappa\, h_{\mu\nu}(x)$$

Expand around background

Full covariance w.r.t. $\bar{g}$

Gauge invariance

$$x^\mu \to x^{\mu\prime} = x^\mu + \xi^\mu$$

$$g'_{\mu\nu} = g_{\mu\nu} - g_{\alpha\nu}\partial_\mu \xi^\alpha - g_{\alpha\mu}\partial_\nu \xi^\alpha + \xi^\alpha \partial_\alpha g_{\mu\nu}$$

$$h'_{\mu\nu} = h_{\mu\nu} \mp \bar{D}_\mu \xi_\nu \mp \bar{D}_\nu \xi_\mu$$

w.r.t. $\bar{g}_{\mu\nu}$

covariant

Grav + ghost loop

$$\Delta \mathcal{L} = \frac{1}{16\pi^2}\left[\frac{1}{\epsilon} + \dots\right]\left\{\frac{1}{120} R^2 + \frac{7}{120} R_{\mu\nu} R^{\mu\nu}\right\}$$

renorm coeff.

Limits of EFT

$G c_i q^2$

$$\mathcal{M} = \mathcal{M}_0 \left[1 + G q^2 \ln q^2 + G^2 q^4 + \dots \right]$$

Fails $\quad G q^2 \sim \mathcal{O}(1) \qquad q^2 > M_P^2$

Fails $\quad G c_i q^2 > 1$

IR issue:

Grav interactions build up

$$\underbrace{\frac{1}{1 - \frac{GM}{r}}}_{\text{large classical effect}}$$

large classical effect — Diff eq

Only weak field quantum

Techniques

BH issues

Core theory:

$$Z = \int_{\Lambda} [d\phi\, dA\, d\psi\, dg_{\mu\nu}] \exp i \int d^4x \sqrt{-g} \Big[-\frac{1}{4}F^2 + \bar{\psi} i \not{D} \psi + \partial_\mu \phi^* \partial^\mu \phi - V(\phi)$$
$$- \Lambda - \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \Big]$$

"A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data."

Frank Wilczek
Physics Today
2002