Nordic 3 GR as QFT/EFT Donoghue JAN 5, 2019

Note Title

EFT is ideal for quantum gravity

EFT resolves apparent problem of QM $\mathcal{M}_{if} = \underbrace{\sum_I}_{\text{all!}} \frac{\langle f|V|I\rangle\langle I|V|i\rangle}{E - E_I}$

ALL physics from high energy appears local

EFT has most general local $\mathcal{L}$

Gravity is unknown in UV, but known at low E

GR has massless gravitons => long range/non-local

EFT is a full QFT

EFT rules

1) Identify Low energy D.O.F.

2) Most general $\mathcal{L}$, ordered by energy expansion

3) Quantize lowest order and start calculating

4) Renormalize relevant parameters (power counting informs this)

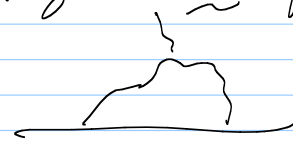
5) Match/measure parameters

6) Predictions from low energy propagation

What are the quantum predictions?

- not divergences
- not parameters
- relations   mostly symmetry
- non-analytic in q^2 $\sim \sqrt{-q^2}\,,\ \ln(-q^2)$

nonlocal



$$g = \eta + \kappa h$$

$h\,\partial^2 h \qquad\qquad \kappa^2 h\,\partial^4 h$

$$S = \int d^4x \sqrt{-g}\left[\cancel{-\Lambda} + \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \cdots\right]$$

$\kappa^2 = 32\pi G_N \qquad\qquad c < 10^{65}$

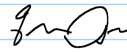
$$D \sim \frac{1}{q^2 + G c_i q^4} = \frac{1}{q^2} - \frac{1}{q^2 + 1/Gc_i}$$

Yukawa $m = \frac{1}{\sqrt{Gc_i}} = \frac{M_P}{c_i}$

$$= \frac{1}{q^2} + \frac{1}{q^2}\, G c_i q^4 \frac{1}{q^2} + \cdots$$

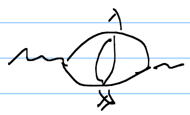
$c_i\, \delta^3(x)$ at low E

Power counting.

Scalar  $\int \frac{d^4 l}{(2\pi)^4} \frac{\kappa}{2}(l_\mu l_\nu + g \cdots) \frac{i}{l^2} \frac{i}{(l+q)^2} \frac{\kappa}{2}[l\, l - q]$

$\sim G(q\,q\,q\,q)\cdots$

$\sim R^2$, c_1, c_2

add $\lambda\phi^4$  also $O(q^4)$

Gravity One loop q^4

2 loop q^6

$\vdots$

Scalar:

1) Do it $G \frac{1}{16\pi^2}\left[\frac{1}{\epsilon} \cdots + \ln q^2\right](q\,q\,q\,q)\cdots$

2) BFM g = BF

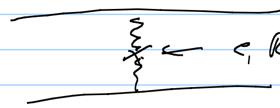
$$\delta S_{Div} = \frac{1}{16\pi^2}\frac{1}{180}\left[\frac{1}{\epsilon} + \cdots\right]\left[3R_{\mu\nu}R^{\mu\nu} - R^2\right]$$

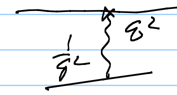
$$\mathcal{L} = d_1 R_{\mu\nu}\partial^\mu\phi\,\partial^\nu\phi$$

EFT reasoning

$V(r)$

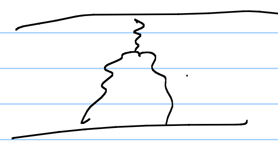
1) No divergences
   No parameters





$$\frac{1}{q^2}\, q^4\, \frac{1}{q^4} \longrightarrow V \sim \delta^3(x)$$

2) Non local, Non analytic
   ↑ finite



What to expect

$$\mathcal{M} = \frac{GMm}{q^2}\left[1 + aG(M+m)\sqrt{-q^2} + bGq^2\ln q^2 + cGq^2\right]$$

FT $\frac{1}{q^2} \Rightarrow \frac{1}{r}$, FT $\frac{1}{\sqrt{q^2}} \sim \frac{1}{r^2}$, FT $\ln q \sim \frac{1}{r^3}$

FT $1 = \delta^3(x)$

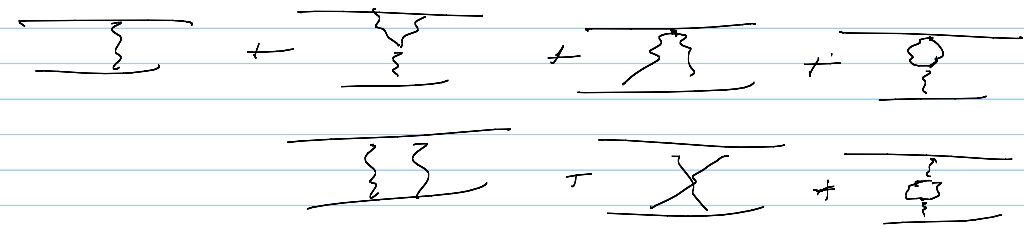
$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{\mathbf{q}^2} = \frac{1}{4\pi r},$$
$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2},$$
$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = -\frac{1}{2\pi r^3},$$

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + c_i\delta^3(x)$$

↑ Classical   ↑ quantum

Bjerrum Bohr,
JFD, Holstein
Kryplovich Kirilin

Calculations



$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

classical   quantum
low energy theorem

EFT pure gravitons are one loop finite

$$R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu}$$

Low order $R_{\mu\nu}=0$ vanish by eq of m
"redundant"

Graviton - Graviton Scattering

$$iM_{tree}(++;++) = \frac{i}{4}\kappa^2 \frac{s^3}{tu}\,,$$

Dunbar + Norridge

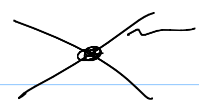
$$
\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\,\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\,\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-u)}{ts}\right)\right]
\end{aligned}
$$

$\frac{2}{\epsilon_{IR}}$

where

$$
\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}
$$

Torma



$1/\varepsilon$ are IR divergences, cured by IR radiation

$$
\begin{aligned}
&\left(\frac{d\sigma}{d\Omega}\right)_{tree}+\left(\frac{d\sigma}{d\Omega}\right)_{rad.}+\left(\frac{d\sigma}{d\Omega}\right)_{nonrad.}= \qquad (29)\\
&=\frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1+\frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s}+\frac{tu}{2s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\right.\\
&\left.\left.-\left(\frac{t}{s}\ln\frac{-t}{s}+\frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2)+\gamma+\ln\frac{s}{\Lambda^2}+\frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.
\end{aligned}
$$

IR

Low energy thm.