EXERCISES

Note Title 7/10/2018

1) **U(1) effective lagrangian**

Consider a theory with a complex scalar field φ with a $U(1)$ global symmetry $\varphi \to \varphi' = \exp{(i\theta)}\ \varphi$. The lagrangian will be

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$$

a) Minimize the potential to find the ground state and write out the lagrangian in the basis

$$\varphi = \frac{1}{\sqrt{2}}(v + \varphi_1(x) + i\varphi_2(x))$$

Show that φ_2 is the Goldstone boson.

b) Use this lagrangian to calculate the low-energy scattering of $\varphi_2 + \varphi_2 \to \varphi_2 + \varphi_2$. Show that despite the non-derivitive interactions of the lagrangian, cancelations occur such that leading scattering amplitude starts at order p^4.

c) Instead of the basis above express the lagrangian using an exponential basis,

$$\varphi = \frac{1}{\sqrt{2}}(v + \Phi(x))e^{i\chi(x)/v} \quad .$$

Show that in this basis a 'shift symmetry' $\chi \to \chi + c$ is manifest.

d) Calculate the same scattering amplitude using this basis and show that the results agree. Note that the fact that the amplitude is of order p^4 is more readily apparent in this basis.

Renaming fields (Haag's thm.)

1) **Effective lagrangian for $\boldsymbol{\mu \to e + \gamma}$**
In describing the decay $\mu \to e + \gamma$, one may try to use an effective lagrangian $\mathcal{L}_{3,4}$ which contains terms of dimensions 3 and 4,

$$\mathcal{L}_{3,4} = a_3(\bar{e}\mu + \bar{\mu}e) + ia_4(\bar{e}\,\not{D}\mu + \bar{\mu}\,\not{D}e)$$

where $D_\mu \equiv \partial_\mu + ieQ_{\rm el}A_\mu$ and a_3, a_4 are constants.

a) Show by direct calculation that $\mathcal{L}_{3,4}$ does *not* lead to $\mu \to e + \gamma$.

b) If $\mathcal{L}_{3,4}$ is added to the QED lagrangian for muons and electrons, show that one can define new fields μ' and e' to yield a lagrangian which is diagonal in flavor. Thus, even in the presence of $\mathcal{L}_{3,4}$, there are *two* conserved fermion numbers.

c) At dimension 5, $\mu \to e + \gamma$ can be described by a gauge-invariant effective lagrangian containing constants c, d,

$$\mathcal{L}_5 = \bar{e}\sigma^{\alpha\beta}(c + d\gamma_5)\mu F_{\alpha\beta} + \text{h.c.} \quad .$$

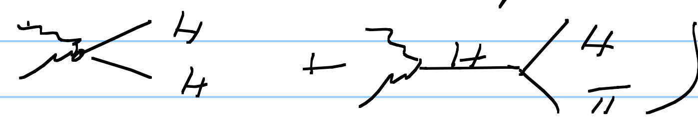
Obtain bounds on c, d from the present limit for $\mu \to e + \gamma$.

Effective Lagrangian for Higgs

In class we derived an approximate effective Lagrangian for the Higg coupling to gauge fields

$$\mathcal{L}_{eff} = \frac{\alpha_s}{24\pi} \ln\left(\frac{v+H}{v}\right) F^a_{\mu\nu} F^{a\mu\nu}$$

(valid when $m_H \ll M_t$).

Use this to show that the $GG \to HH$ amplitude vanishes at threshold (Hint 

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This is phenomenologically relevant as it is an approx. reason for the suppressed di-Higgs rate.

Gravity without tensor indices

Using $\mathcal{L} \sim \left[\frac{1}{G} R + c R^2\right]$ and dropping indices on the metric $(g = 1 + h)$:

1) Treating both terms on equal footing, find the propagator

2) Show that the potential is $\left(\frac{1}{r} + \text{Yukawa}\right)$

3) Find a bound on c, knowing that table top expts do not see the Yukawa

4) The EFT Treatment has a $\delta^3(x)$ instead of a Yukawa. How is this consistent?

Schroedinger eq for gravity

Take a scalar field coupled to gravity

$$\sqrt{-g}\,\mathcal{L}^{\phi} = \sqrt{-g}\left[g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi - m^2 \phi^2 \right]$$

with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$; $h_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} \dfrac{2GM}{r}$

Find the Euler Lagrange equations, take the non relativistic limit and thereby find the Schroedinger eq in a gravitational field.

Heat kernel exercise

Use the a_2 coefficient given in class to obtain the wavefunction renormalization constant for QED. [The graft web page has the Appendix B from "Dynamics of the Standard Model" which also contains the relevant formulas.]