Lecture 4

13 October 2016

1 General Relativity as an effective field theory

In the previous lecture we have learned how effective field theory works. Now we can straightforwardly apply these ideas to General Relativity (GR) and see that it perfectly fits into the effective field theory description. Technically, all interaction vertices of GR are energy-dependent and thus effortlessly organize an EFT energy expansion. The GR interactions are non-renormalizable, and the suppression scale is given by the Planck mass $\sim 10^{18}$ GeV. The shortest scales at which gravity can be directly tested are several tens of micrometers [1], which corresponds to energy ~ 0.1 eV. The energies accessible at LHC are about 10 TeV, while the most energetic cosmic rays were detected at 10^{11} GeV. The highest energy scale accessible in principle (at this moment) is the scale of inflation equal to 10^{16} GeV at most [2]. Clearly, all these scales are well below the Planck energy which serves as a cutoff in GR if treated as an EFT.[1] Thus, from the phenomenological point of view, GR should be enough to account for effects of quantum gravity within the EFT framework. In this lecture we will apply one by one the EFT principles listed in Lecture 3 to GR and show that quantum gravity is indeed a well-established and predictive theory.

1.1 Degrees of freedom and interactions

As a first step we identify low energy degrees of freedom and their interactions. These are the helicity-2 transverse-traceless graviton and matter fields (in this lecture represented by a real scalar ϕ).

1.2 Most general effective Lagrangian

Let us go for the steps (2) and (3). The most general Lagrangian for gravity which is consistent with diffeomorphisms and local Lorentz transformations

[1] Formally, the cutoff of GR may depend on the number of matter degrees of freedom which can run into gravity loops.

takes the following form, if ordered in the energy expansion,

$$S = \int d^4x\sqrt{-g}\left[-\Lambda - \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + ...\right] . \qquad (1)$$

Recall that $R \sim \partial^2 g$, where g denotes the metric, so the leftmost term (cosmological constant) is $O(E^0)$, the second - $O(E^2)$ and the c_i terms scale as $O(E^4)$ in the energy expansion.[2] The most generic local energy-ordered effective Lagrangian for matter takes the following form,

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Bigg[& - V(\phi) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \xi\phi^2R \\ & + \frac{d_1}{M_P^2}Rg^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{d_2}{M_P^2}R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + ...\Bigg] , \end{aligned} \qquad (2)$$

with dimensionless couplings ξ, d_1, d_2. For the sake of simplicity we will put these parameters to zero in what follows and focus only on the minimal coupling between gravity and matter.

1.3 Quantization and Renormalization

At the step (4) we should begin to calculate starting with the lowest order. In fact, we have already computed the 1-loop effective action in Lecture 2. The result is,

$$\Delta\mathcal{L}_{div.} = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\ 4\pi - \gamma\right)\left[\left\{\frac{1}{120}R^2 + \frac{7}{120}R_{\mu\nu}R^{\mu\nu}\right\} + \frac{1}{180}\left(3R_{\mu\nu}R^{\mu\nu} - R^2\right)\right] , \qquad (3)$$

where the terms inside the curly brackets come from graviton loops and the terms inside the round brackets come from the matter loops. Then, we renormalize the couplings as follows,

$$\begin{aligned} c_1^{\bar{M}S} &= c_1 + \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\ 4\pi - \gamma\right)\left[\frac{1}{120} - \frac{1}{60}\right] , \\ c_2^{\bar{M}S} &= c_2 + \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\ 4\pi - \gamma\right)\left[\frac{7}{120} + \frac{3}{60}\right] . \end{aligned} \qquad (4)$$

1.4 Fixing the EFT parameters

The EFT parameters Λ, κ^2, c_i are to be measured experimentally (step (6) in our program).

[2]Notice that we are working in four dimensions and assume trivial boundary conditions, which, by virtue of the Gauss-Bonnet identity (see Lecture 2), allows us to eliminate from the action another curvature invariant, $R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$.

1) The cosmological constant is believed to be responsible for the current acceleration expansion of the Universe. This hypothesis is consistent with all cosmological probes so far, and the inferred value of the cosmological constant is

$$\Lambda \simeq 10^{-47}\ (\mathrm{GeV})^4\,. \tag{5}$$

The cosmological constant has a very tiny effect on ordinary scales and is negligible for practical computations as long as we work at distances shorter than the cosmological ones. In what follows we will assume that the cosmological constant is zero.

2) The parameter κ^2 defines the strength of gravitational interactions at large scales. Neglecting for a moment the c_i terms, the tree-level gravitational potential of interaction between two point masses m_1 and m_2 takes the form,

$$V(r) = -\frac{\kappa^2}{32\pi}\frac{m_1 m_2}{r}\,, \tag{6}$$

from which one deduces the relation to the Newtonian gravitational constant,

$$\kappa^2 = 32\pi G\,. \tag{7}$$

3) The constants c_i produce Yukawa-type corrections to the gravitational potential which become relevant at distances $\sim \kappa\sqrt{c_i}$. Indeed, taking into account the c_i terms one can obtain the tree-level gravitational potential of the form [3],

$$V(r) = -\frac{\kappa^2}{32\pi}\frac{m_1 m_2}{r}\left(1 + \frac{1}{3}e^{-M_1 r} - \frac{4}{3}e^{-M_2 r}\right)\,, \tag{8}$$

where

$$\begin{aligned} M_1^2 &\equiv \frac{1}{(3c_1 + c_2)\kappa^2} \\ M_2^2 &\equiv -\frac{2}{c_2\kappa^2}\,. \end{aligned} \tag{9}$$

The laboratory tests of gravity at short scales imply

$$|c_i| < 10^{56}\,. \tag{10}$$

In order to understand the above results let us focus on a toy model of gravity without tensor indices.

1.4.1 Gravity without tensor indices

Consider the following action

$$S = \int d^4x\sqrt{-g}\left(-\frac{2}{\kappa^2}R + cR^2\right)\,. \tag{11}$$

Expanding the toy metric g as follows,

$$g = 1 + \kappa h \,, \tag{12}$$

one arrives at the following free equation of motion for the "graviton",

$$(\Box + c\kappa^2 \Box^2) h = 0 \,. \tag{13}$$

The propagator then takes the form,

$$D(q^2) = \frac{1}{q^2 + c\kappa^2 q^4} \equiv \frac{1}{q^2} - \frac{1}{q^2 + (\kappa^2 c)^{-1}} \,. \tag{14}$$

Then we couple the "scalar graviton" to matter,

$$S_m = \frac{1}{2} \int d^4x \sqrt{-g} \left(g(\partial\phi)^2 - m^2\phi^2 \right) \,, \tag{15}$$

and compute the tree-level gravitational potential. Introducing the notation $M^2 \equiv (\kappa^2 c)^{-1}$ we perform a Fourier transform to finally get

$$V(r) = -\frac{Gm_1m_2}{r}(1 - e^{-Mr}) \,. \tag{16}$$

The current laboratory constraint on the Yukawa-type interactions imply the bound

$$M < 0.1 \text{ eV} \quad \Rightarrow \quad c < 10^{56} \,. \tag{17}$$

An important observation can be made by taking the limit $M \to \infty$, in which the Yukawa part of the potential reduces to a representation of the Dirac delta-function,

$$\frac{1}{4\pi r} e^{-Mr} \to \frac{1}{M^2} \delta^{(3)}(\mathbf{x}) \,. \tag{18}$$

Thus, the gravitational potential from Eq. (16) can be rewritten as

$$V(r) = -\frac{Gm_1m_2}{r} + c\, G^2 \delta^{(3)}(\mathbf{x}) \,. \tag{19}$$

This expression reminds us of local quantum correction related to divergent parts of loop integrals. In fact, this result merely reflects the fact that $\sim R^2$ terms are generated by loops.

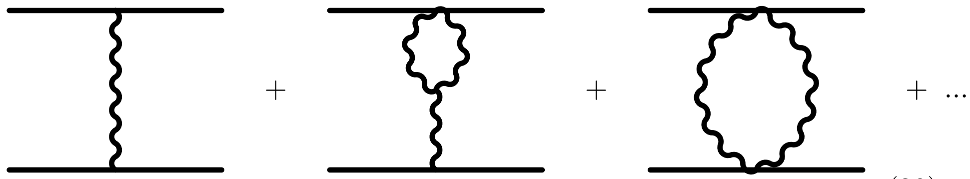
A comment is in order. The fact that the propagator of the higher-order theory (14) can be cast into the sum of two "free" propagators suggests that there are new degrees of freedom that appear if we take into account the $\sim R^2$ terms. In fact, non-zero c_i lead to the appearance of a scalar DOF of mass M_1 (see Eq. (9)) and a massive spin-2 DOF of mass M_2.

1.5 Predictions: Newtonian gravitational potential at one loop

So far we have made no predictions. We performed renormalization and measured (constrained) the free parameters of our EFT. As we learned from the example of the sigma-model, the most important predictions of the EFT are related to non-analytic in momenta loop contributions to the interaction vertices. They are typically represented by logarithms and correspond to long-range interactions induced by virtual particles.[3]

In this subsection we will demonstrate the Newtonian potential at one loop and show that the predictions of GR treated as an EFT are no qualitatively different from that of the sigma-model.

At one-loop order there appear a lot of diagrams contributing to the gravitational potential. Here is a very incomplete sample of them,



(20)

From the power counting principles we anticipate that the 1-loop amplitude will take the following form,

$$\mathcal{M} = \frac{Gm_1m_2}{q^2}\left(1 + aG(m_1+m_2)\sqrt{-q^2} + b\,Gq^2\ln(-q^2) + c\,Gq^2\right), \quad (21)$$

where a, b, c are some constants. Then, assuming the non-relativistic limit and making use of

$$\begin{aligned} \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{\mathbf{q}^2} &= \frac{1}{4\pi r}, \\ \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} &= \frac{1}{2\pi^2 r^2}, \\ \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) &= -\frac{1}{2\pi r^3}, \end{aligned} \quad (22)$$

we recover the following potential in position space:

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + a\frac{G(m_1+m_2)}{r} + b\frac{G}{r^2}\right) + c\,G\delta^{(3)}(\mathbf{x})\,. \quad (23)$$

The delta-function term is irrelevant as it does not produce any long-distance effect. The a and b terms are relevant though. By dimensional analysis we

[3] Note that in renormalizable field theories the effect of non-analytic contributions can be interpreted as running of coupling constants with energy.

can restore the speed of light c_l and the Planck constant $\hbar$ in the expression for them,

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + a\frac{G(m_1+m_2)}{rc_l^2} + b\frac{G\hbar}{r^2c_l^3}\right) . \tag{24}$$

The a-term thus represents a classical correction that appears due to the non-linearity of GR while the b-term is a quantum correction.

An explicit calculation has been carried out in Ref.[4] and gives

$$\begin{aligned} a &= 3 \,, \\ b &= \frac{41}{10\pi} \,. \end{aligned} \tag{25}$$

The c_i terms in our EFT expansion give only local contributions $\sim \delta^{(3)}(\mathbf{x})$ and thus can be dropped. The result (24) with the coefficients (25) should be true in any UV-completion of gravity that reduces to GR in the low-energy limit. The quantum correction (b-term) is extremely tiny and scales as $(l_P/r)^2$ in full agreement with the EFT logic.

As for the classical correction (a-term), it agrees with the Post-Newtonian expansion in a proper coordinate frame. Quite unexpectedly, this correction came out of the loop calculation even though one might have thought that loop corrections should scale as powers of $\hbar$. This is not true, and we can demonstrate an even simpler example of that. Consider the action for a fermion in flat spacetime,

$$S = \int d^4x \, \bar{\psi} \left(\not{D} - m \right) \psi \,. \tag{26}$$

Introducing $\hbar$ and c_l this action can be rewritten as,

$$S = \hbar \int d^4x \, \bar{\psi} \left(\not{D} - \frac{mc_l^2}{\hbar} \right) \psi \,. \tag{27}$$

One observes the appearance of $\hbar$ in the denominator, which can cancel some $\hbar$ coming from loops and eventually result in a classical correction.

2 Generation of the Reissner-Nordström metric through loop corrections

Another instructive example showing EFT ideas at work is the calculation of quantum corrections to the Reissner-Nordström metric (static spherically-symmetric GR solution for charged point objects), see [5] for more detail. In this case dominating quantum corrections are produced by matter fields running inside loops, the metric can be treated as a classical field. The classical metric couples to the energy momentum tensor of matter, whose quantum fluctuations induce corrections to the metric. The net result in the harmonic gauge reads,

$$\begin{aligned} g_{00} &= 1 - \frac{2GM}{r} + \frac{G\alpha}{r^2} - \frac{8G\alpha}{3\pi M r^3}\,, \\ g_{ij} &= \delta_{ij}\left(1 + \frac{2GM}{r}\right) + \frac{G\alpha n_i n_j}{r^2} + \frac{4G\alpha}{3\pi M r^3}\left(n_i n_j - \delta_{ij}\right)\,, \end{aligned} \tag{28}$$

where

$$\begin{aligned} \alpha &= \frac{e^2}{4\pi}\,, \\ n_i &\equiv \frac{x_i}{r}\,. \end{aligned} \tag{29}$$

We start by considering a charged scalar particle on the flat background. As shown in Lecture 1, in the harmonic gauge the Einstein equation for a metric perturbation $h_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \tag{30}$$

takes the following form:

$$\Box h_{\mu\nu} = -8\pi G(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^{\lambda}_{\lambda})\,. \tag{31}$$

Assuming a static source, upon introducing the retarded Green's function we obtain,

$$h_{\mu\nu} = -8\pi G \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{qx}} \frac{1}{\mathbf{q}^2}(T_{\mu\nu}(\mathbf{q}) - \frac{1}{2}\eta_{\mu\nu}T^{\lambda}_{\lambda}(\mathbf{q}))\,. \tag{32}$$

The energy momentum tensor is a quantum variable. In what follows we assume that the matter is given by a scalar field of mass m, which is coupled to photons. The transition density takes the form

$$\langle p'|T_{\mu\nu}|p\rangle = \frac{e^{i(p'-p)x}}{\sqrt{2E2E'}}\left[2P_\mu P_\nu F_1(q^2) + (q_\mu q_\nu - \eta_{\mu\nu}q^2)F_2(q^2)\right]\,, \tag{33}$$

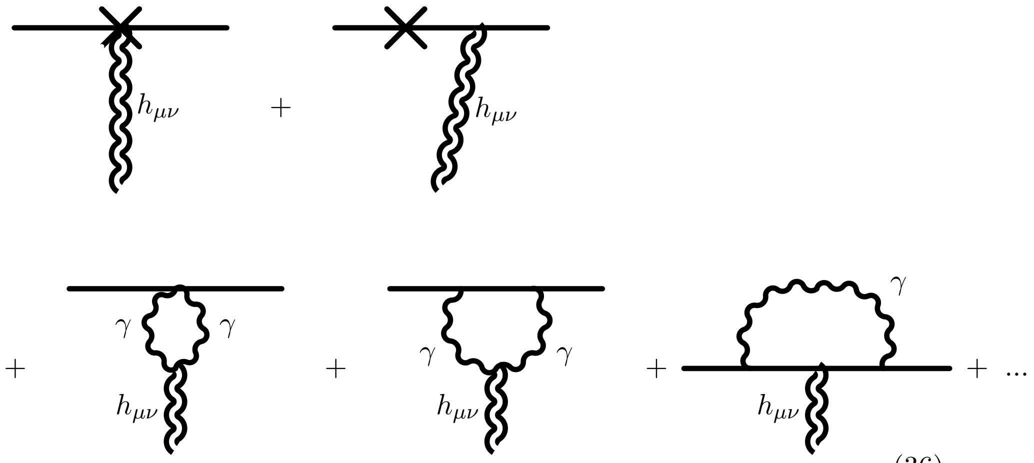
where

$$P_\mu \equiv \int d^3x T_{0\mu}\,. \tag{34}$$

At tree level one has,

$$\begin{aligned} F_1(q^2) &= 1\,, \\ F_2(q^2) &= -\frac{1}{2}\,. \end{aligned} \tag{35}$$

The radiative corrections to $T_{\mu\nu}$ are given by the following diagrams



(36)

The form-factors in the limit $q \to 0$ read,

$$F_1(q^2) = 1 + \frac{\alpha}{4\pi}\frac{q^2}{m^2}\left(-\frac{8}{3} + \frac{3}{4}\frac{m\pi^2}{\sqrt{-q^2}} + 2\ln\frac{-q^2}{m^2}\right) ,$$

$$F_2(q^2) = -\frac{1}{2} + \frac{\alpha}{4\pi}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{m^2}{4\pi\mu^2} - \frac{26}{9} + \frac{m\pi^2}{2\sqrt{-q^2}} + \frac{4}{3}\ln\frac{-q^2}{m^2}\right) . \tag{37}$$

The classical corrections $\sim \sqrt{-q^2}$ come only from the middle diagram of the last line in Eq. (36), while the "quantum" logarithms are produced by both the left and the middle diagrams of the last line in Eq. (36).

Let us comment more on the origin of the classical terms. In position space the energy-momentum tensor takes the form,

$$\begin{aligned} T_{00} &= m\delta^{(3)}(\mathbf{x}) + \frac{\alpha}{8\pi r^4} - \frac{\alpha}{\pi^2 m r^5} , \\ T_{ij} &= -\frac{\alpha}{4\pi r^4}\left(n_i n_j - \frac{1}{2}\delta_{ij}\right) - \frac{\alpha}{3\pi^2 m r^5}\delta_{ij} . \end{aligned} \tag{38}$$

This should be compared with the expression for the energy-momentum tensor of the electromagnetic field around a static charged particle,

$$\begin{aligned} T^{EM}_{\mu\nu} &= -F_{\mu\lambda}F_{\nu}{}^{\lambda} + \frac{1}{4}\eta_{\mu\nu}F^2_{\alpha\beta} , \\ T^{EM}_{00} &= \frac{\vec{E}^2}{2} = \frac{\alpha}{8\pi r^4} , \\ T^{EM}_{ij} &= -E_i E_j + \delta_{ij}\frac{\vec{E}^2}{2} = -\frac{\alpha}{4\pi r^4}\left(n_i n_j - \frac{1}{2}\delta_{ij}\right) . \end{aligned} \tag{39}$$

One concludes that the classical corrections just represent the electromagnetic field surrounding the charged particle. These corrections reproduce the classical Reissner-Nordström metric and are required in order to satisfy the Einstein's equations.

Thus, starting from a charged particle on the flat background, we computed loop corrections to the metric, which yielded the classical Reissner-Nordström metric plus a quantum correction.

3 GR as EFT: further developments

3.1 Gravity as a square of a gauge theory

We started our Lectures by constructing GR in the gauge theory framework. We saw that there is deep connection between gravity and YM-theories. Here we want to explore this connection from different perspective. Meditating on immense complexity of quantum gravity amplitudes, it is tempting to search for their relation to YM-amplitudes, since the calculation of the latter is incomparably easier. Observing that the graviton field $h_{\mu\nu}$ has a meaning of the tensor product of two vector objects, one may guess that

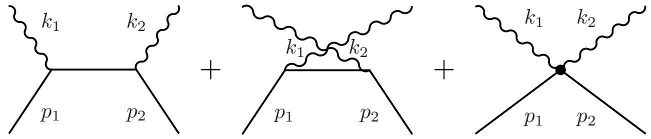
$$\text{gravity} \quad \sim \quad \text{gauge theory} \times \text{gauge theory}\,. \tag{40}$$

The question of how to endow this intuitive statement with precise meaning is far from being obvious. The answer comes from string theory, where there are so-called Kawai-Lewellen-Tye (KLT) relations that connect closed and open string amplitudes [6]. Since closed strings correspond to gravitons, and open strings correspond to gauge bosons, these relations must link quantum gravity amplitudes to YM-amplitudes in the field theory limit. The KLT-relations provide us with the desired simplification in computing the diagrams in quantum gravity.

To understand why the KLT-relations actually take place within the field theory framework, it is desirable to derive them without appealing to string theory. Speaking loosely, one should "decouple" the left and right indices of $h_{\mu\nu}$ in order to associate a gauge theory to each of them. Taking GR as it is, we see that such decoupling is not achieved even at quadratic order in κ, in particular due to plenty of h^{μ}_{μ} pieces (see Lecture 2). An elaborate procedure of redefining the fields must be implemented before this becomes possible. For further details, see [7].

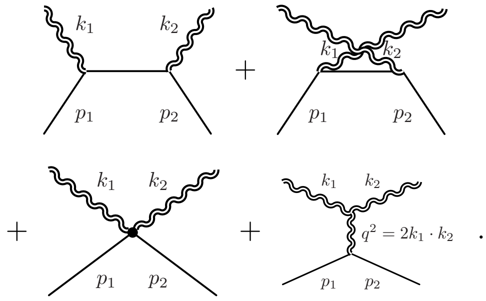
As an instructive example of the application of the KLT-relations, consider the gravitational Compton scattering process. Namely, let $\phi^{(s)}$ be massive spin$-s$ matter field, $s = 0, \frac{1}{2}, 1$, with mass m. Consider the QED with the field $\phi^{(s)}$ coupled to the photon field in the usual way, and let e denote the coupling constant. The tree-level scattering process in QED is described by the following sum of diagrams,

$$i\mathcal{M}^{(s)}_{EM}(p_1,p_2,k_1,k_2) =$$



Here we use the "all-incoming" notation for momenta, so that $p_1+p_2+k_1+k_2=0$. On the other hand, the gravitational scattering amplitude is represented by the series of diagrams

$$i\mathcal{M}^{(s)}_{grav.}(p_1,p_2,k_1,k_2) =$$



To understand the enormous difficulty of the straightforward calculation of this amplitude, one can just recall the general expression for the tree-graviton vertex quoted in [8]. It makes truly remarkable the fact that $\mathcal{M}^{(s)}_{grav.}$ is actually equal to [9]

$$\mathcal{M}^{(s)}_{grav.}(p_1,p_2,k_1,k_2) = \frac{\kappa^2}{8e^2}\frac{(p_1\cdot k_1)(p_1\cdot k_2)}{(k_1\cdot k_2)}\mathcal{M}^{(s)}_{EM}(p_1,k_2,p_2,k_1) \times\mathcal{M}^{(0)}_{EM}(p_1,k_2,p_2,k_1)\,. \tag{41}$$

Let us take $s=0$ for simplicity. Then, using the Helicity formalism notations of Ref.[10], the amplitude (41) can be brought to the form

$$\begin{aligned} i\mathcal{M}^{(0)}_{grav.}(p_1,p_2,k_1^+,k_2^+) &= \frac{\kappa^2}{16}\frac{m^4[k_1k_2]^4}{(k_1\cdot k_2)(k_1\cdot p_1)(k_1\cdot p_2)}\,,\\ i\mathcal{M}^{(0)}_{grav.}(p_1,p_2,k_1^-,k_2^+) &= \frac{\kappa^2}{16}\frac{\langle k_1|p_1|k_2]^2\langle k_1|p_2|k_2]^2}{(k_1\cdot k_2)(k_1\cdot p_1)(k_1\cdot p_2)}\,, \end{aligned} \tag{42}$$

and

$$\begin{aligned} i\mathcal{M}^{(0)}_{grav.}(p_1,p_2,k_1^-,k_2^-) &= (i\mathcal{M}^{(0)}_{(grav)}(p_1,p_2,k_1^+,k_2^+))^*\,,\\ i\mathcal{M}^{(0)}_{grav.}(p_1,p_2,k_1^+,k_2^-) &= (i\mathcal{M}^{(0)}_{(grav)}(p_1,p_2,k_1^-,k_2^+))^*\,. \end{aligned} \tag{43}$$

Here we denote by k_i^+ the $(++)$ polarization of the graviton, and by k_i^- — its $(--)$ polarization.

3.2 Loops without loops

Now we want to make one step further and see how one can simplify the computation of loop diagrams in quantum gravity. A natural method here is to use the Optical theorem. Making use of the unitarity of S-matrix, $S^\dagger S = 1$, where $S = 1 + iT$, we have

$$2\mathrm{Im}T_{if} = \sum_j T_{ij}T^\dagger_{jf}\,. \tag{44}$$

In this expression, i and f denote initial and final states respectively, and the sum is performed over all intermediate states. Eq.(44) allows us to express the imaginary part of 1-loop diagrams in terms of tree-level diagrams. The reconstruction of the whole loop amplitude from its imaginary part can be tackled in several ways. The traditional way is to use dispersion relations. In general this method has unpredictable subtraction constants in the real part of the amplitude, which cannot be eliminated. However, the non-analytic corrections are independent of the subtraction constants and are predictable.

A more modern way is to explore unitarity in the context of dimensional regularization. It turns out that there are large classes of 1-loop amplitudes in various theories, that can be uniquely reconstructed from tree diagrams by using the $D-$dimensional unitarity technique. Any such amplitude can be represented as $\mathcal{M} = \sum_i c_i I_i$, where c_i are rational functions of the momentum invariants and I_i are some known integral functions representing sample 1-loop diagrams (these include box, triangle and bubble integrals). It can be proven that if two linear combinations $\sum_i c_i I_i$ and $\sum_i c'_i I_i$ coincide on cuts, then they must coincide everywhere [4].

For many practical purposes there is no need for the reconstruction of the whole 1-loop amplitude. For example, consider the diagram presented in Fig.1. It provides a quantum correction to the Coulomb potential or to the Newton's potential. Cutting it as demonstrated in Fig.1, one can express its imaginary part in terms of the corresponding tree diagrams. This imaginary part contains enough information to extract non-analytic

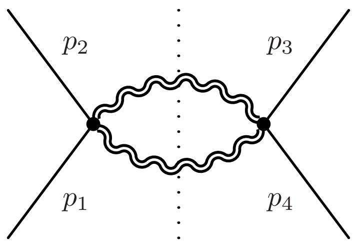


Figure 1: 1-loop diagram providing quantum corrections to the Coulomb or Newton's potential. Dotted line represents the cut.

contributions to the quantum correction like the classical contribution from GR and the quantum gravity contribution to the Newton's potential. The essential features of such calculation are

[4]For further discussion, see [11].

- we impose on-shell cut condition everywhere in the numerator,
- the computation does not require any ghost contributions,
- the non-analytic terms give us leading *long-ranged* corrections to the potential.

3.3 An application: bending of light in Quantum gravity

Let us briefly demonstrate how to apply the tools we have just described to a real computation. Consider the light bending in quantum gravity and calculate the long-ranged quantum correction to the deflection angle of a beam of massless particles (scalars or photons) as they scatter off a massive scalar object (like the Sun) of mass M. Our strategy is the following [12],

- write the tree-level QED Compton amplitudes,
- express the gravitational tree-level Compton amplitude through the corresponding QED amplitudes,
- write the discontinuity of the gravitational 1-loop scattering amplitude in terms of the on-shell tree-level amplitudes,
- from this discontinuity, extract the power-like and logarithm corrections to the scattering amplitude,
- compute the potential in the Born approximation and deduce the bending angle of a photon and for a massless scalar.

We have already given most of the results of the first two points of this program. The tree-level massive scalar-graviton interaction amplitudes are given by Eqs.(42),(43). Let us quote the result for the photon-graviton interaction amplitude,

$$i\mathcal{M}^{(1)}_{grav.}(p_1^+, p_2^-, k_1^+, k_2^-) = \frac{\kappa^2}{4}\frac{[p_1k_2]^2\langle p_2k_2\rangle^2\langle k_2|p_1|k_1]^2}{(p_1\cdot p_2)(p_2\cdot k_1)(p_1\cdot k_2)}\,. \tag{45}$$

As for the other helicities, $i\mathcal{M}^{(1)}_{grav.}(p_1^-, p_2^+, k_1^+, k_2^-)$ is obtained from the expression above by the momenta p_1 and p_2 interchanged, and amplitudes with opposite helicity configurations are obtained by complex conjugation.

Let us turn to the third point of the program. The 1-loop diagram responsible for our scattering process is presented in Fig.2. We make two gravitons cut and write the discontinuity as follows,

$$i\overset{1}{\mathcal{M}}{}^{(s)}_{grav.}(p_1^{\lambda_1}, p_2^{\lambda_2}, p_3, p_4)\Big|_{\text{disc.}} =$$

$$\int \frac{d^Dl}{(2\pi)^4}\frac{\sum_{h_1,h_2}\mathcal{M}^{(s)}_{grav.}(p_1^{\lambda_1}, p_2^{\lambda_2}, l_1^{h_1}l_2^{h_2})\cdot(\mathcal{M}^{(0)}_{grav.}(l_1^{h_1}, l_2^{h_2}, p_3, p_4))^*}{4l_1^2l_2^2}\,. \tag{46}$$

In this expression, $l_1^2 = l_2^2 = 0$ are the cut momenta of the internal graviton lines, h_i — their polarizations, and λ_i — possible polarizations of the massless particle, $s = 0, 1$, and $D = 4 - 2\epsilon$.

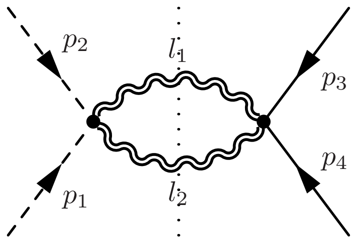


Figure 2: 1-loop diagram providing quantum corrections to the light bending. The dashed lines represent massless field (scalar or photon), the solid line — the massive field, and the dotted line represents the cut.

Now one can compute the full amplitude $i\mathcal{M}^{(s)} = \frac{i}{\hbar}\mathcal{M}^{(s)}_{grav.} + i\overset{1}{\mathcal{M}}{}^{(s)}_{grav.}$. In the low-energy limit, $\omega \ll M$, where ω is the frequency of the massless particle, the leading contribution to $i\mathcal{M}^{(s)}$ is written as [12]

$$i\mathcal{M}^{(s)} \simeq \frac{\mathcal{N}^{(s)}}{\hbar}(M\omega)^2 \times \left[\frac{\kappa^2}{t} + \kappa^4 \frac{15}{512}\frac{M}{\sqrt{-t}} + \right.$$
$$\hbar\kappa^4 \frac{15}{512\pi^2} \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4 \frac{bu^{(s)}}{(8\pi)^2} \log\left(\frac{-t}{\mu^2}\right) +$$
$$\left. \hbar\kappa^4 \frac{3}{128\pi^2} \log^2\left(\frac{-t}{\mu^2}\right) + \kappa^4 \frac{M\omega}{8\pi}\frac{i}{t} \log\left(\frac{-t}{M^2}\right)\right] . \tag{47}$$

Here $\mathcal{N}^{(s)}$ is the prefactor which is equal to 1 for the massless scalar, while for the photon it is given by $\mathcal{N}^{(1)} = (2M\omega)^2/(2\langle p_1|p_3|p_2]^2)$ for the $(+-)$ photon helicity configuration and the complex conjugate of this for the $(-+)$ photon helicity configuration. For $(++)$ and $(--)$ the amplitude vanishes. The coefficient $bu^{(s)}$ equals $3/40$ for the case of the scalar particle and $-161/120$ for the case of photon. Finally, t is the usual kinematic variable.

We can now use Born approximation to calculate the semiclassical potential for a massless scalar and photon interacting with a massive scalar object, and then apply a semiclassical formula or the eikonal approximation [13] for the angular deflection to find for the bending angle

$$\theta^{(s)} \simeq \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8bu^{(s)} + 9 + 48\log\frac{b}{2r_o}}{\pi}\frac{G^2\hbar M}{b^3} . \tag{48}$$

The first two terms give the correct classical values, including the first post-Newtonian correction, expressed in terms of the impact parameter b. The last term is a quantum gravity effect of the order $G^2\hbar M/b^3$. Let us comment on this formula.

- The third contribution in (48) depends on the spin of massless particle scattering on the massive target. Hence the quantum correction is not universal. This may seem to violate the Equivalence Principle. Note, however, that this correction is logarithmic and produces non-local effects. This is to be expected, since for the massless particles quantum effects are not localized, and their description as point particles is not valid anymore. The Equivalence Principle says nothing about the universality of such non-local effects. Anyway, we see that in quantum gravity particles no longer move along the geodesics, and that trajectories of different particles bend differently.

- The answer depends on the IR-scale r_o. However, this does not spoil the predictive power of the theory. For example, one can compare the bending angle of a photon with that of a massless scalar. The answer is

$$\theta^{(1)} - \theta^{(0)} = \frac{8(bu^{(1)} - bu^{(0)})}{\pi}\frac{G^2\hbar M}{b^3}\,. \tag{49}$$

  This result is completely unambiguous. Once again this demonstrates the fact that the EFT technique can make well-defined predictions within quantum gravity.

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