EPFL Lectures

Note Title 10/6/2016

Varieties of EFT

A) Integrating out heavy particles/interactions

- σ model, ChPTh
- BSM
- QG ***

B) Integrating out high F part of a field

- NRQCD, HQET, HBChPTh
- SCET

Exercise

a) Use the QCD treatment which we have done to calculate the contribution to the cosmological constant which is linear in the up quark mass.

b) By what fractional change in m_u would Λ change by 100%?

Solution

$$\mathcal{L}_{QCD} = - - - - \left(\bar{\psi}_L (s + ip) \psi_R + h.c. \right)$$

QCD $\quad s \to m = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \quad , p \to 0$

$$\mathcal{L} = \frac{F^2}{2} \left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \mathrm{Tr}(\chi^\dagger U + \chi U^\dagger) \right]$$

$\chi = 2B_0 (s + ip)$

const

2 parameters $\quad F = 92\,\mathrm{MeV}$

B_0 from $\quad m_\pi^2 = 2B_0 (m_u + m_d)$

$$U = \exp\left(i \vec{\tau} \cdot \vec{\pi} / F \right)$$

$$\langle 0 | \mathcal{L}_{eff} | 0 \rangle = -\Lambda_m = -F^2 2B_0 (m_u + m_d)$$

$$= -F^2 m_\pi^2 = 0.63 \times 10^{43} \Lambda_0$$

$$= 0.63 \times 10^{43} \Lambda_0 \left(\frac{m_u + m_d}{(m_u + m_d)^{phy}} \right)$$

$$\frac{\delta m_u}{m_u} \sim 10^{-43} \Longrightarrow \frac{\delta \Lambda}{\Lambda_0} = 1$$

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Note Title 8/10/2016

General Relativity as an Effective Field Theory

GR fits EFT approach almost perfectly

We have done all of the background work

Now just put it together

Summary of EFT principles:

1) Identify low energy degrees of freedom and symmetry
2) Write most general $\mathcal{L}_{eff}$
3) Order it in the energy expansion $\leftarrow$ **
4) Starting with lowest order, start calculations
5) Renormalize
6) Match or measure $\leftarrow$ C_1, C_2
7) Residual low energy effects are predictions $\boxed{\leftarrow \text{ **}}$

Now to Gravity

Follow EFT rules:

1) Low energy DOF + interactions

graviton
matter } GR

2) Most general Lagrangian consistent with symmetry, (+3) ordered in the energy expansion.

$\swarrow R \sim \partial^2 g$

$$S = \int d^4x \sqrt{g} \left[-\Lambda - \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

$\nwarrow M_P^2$

a) Cosmological constant

- no insight from EFT
- small on ordinary scales

b) $\kappa^2 = 32\pi G_N$

c) $c_1 , c_2 < 10^{+65}$

$$\left(1 + \frac{c_i}{M_P^2} E^2 \right)$$

Gravity without tensor indices

- lets drop all indices
- take $g = 1 + \kappa h$, $\sqrt{g} R = \Box \kappa h + \kappa^2 \partial_\mu h \, \partial^\mu h + \mathcal{O}(h^3)$

$$\sqrt{g}\, \mathcal{L} = \sqrt{-g} \left(\frac{2}{\kappa^2} R + c R^2 \right) + \left(g \partial_\mu \phi \, \partial^\mu \phi - m^2 \phi^2 \right)$$

a) With $c = 0$, calculate one graviton exchange

b) With $c \neq 0$, find propagator and redo one graviton exchange

c) Write the result of b) as that of a) plus a correction. Show that correction is a Yukawa potential in position space.

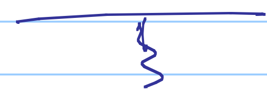
d) Give a rough bound on the parameter c from the fact that gravity tests work at 1 mm

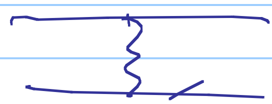
e) A short range Yukawa can be a representation of a Dirac delta function. Find the representation of the correction term as a delta function.

Solution:

$c = 0$

(Stelle) $K \leftarrow \kappa$

a)  $\frac{1}{q^2}$  $= K p_1 \cdot p_2 \sim K m^2$

 $K p_1 \cdot p_2 \frac{1}{q^2} K p_3 \cdot p_4 \sim K \frac{m_1^2 m_2^2}{q^2}$

$\xrightarrow{NR} \frac{1}{\sqrt{2E_1 2E_2 2E_3 2E_4}} \mathcal{M} \xrightarrow{FT} V \sim K^2 \frac{m_1 m_2}{r}$

b) $c \neq 0$ $\qquad R + cR^2 \qquad (\Box + k^2 c\, \Box^2)\, h = 0$

$$\frac{1}{q^2 + k^2 c\, q^4} \rightarrow \frac{1}{q^2} - \frac{1}{q^2 + \frac{1}{k^2 c}} = \frac{1}{q^2} - \frac{1}{q^2 + m^2}$$

$$V(r) = \frac{G m_1 m_2}{r}\left(1 - e^{-mr}\right)$$

GR works at 1 mm $\Rightarrow m > \frac{1}{1\,mm} \Rightarrow c < 10^{65}$

$$\frac{1}{4\pi r} e^{-mr} \longrightarrow \frac{1}{m^2}\,\delta^3(x)$$

$$V(r) = \frac{G m_1 m_2}{r} - G^2 c\, \delta^3(x)$$

Also in matter coupling

$$\mathcal{L} = g^{\mu\nu}\partial_\mu\phi\,\partial^\nu\phi - \xi R\phi^2 + d_1 R\, g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + d_2 R^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$d_1 = \bar{d}_1 / M_P^2$ — dim less

Form of the matrix elements / Feynman rules

$$\langle T_{\mu\nu}\rangle = p_\mu p_\nu + p_\mu p_\nu \frac{\bar{d}_1}{M_P^2} q^2$$

4) Start with lowest order, calculate
5) Renormalize ←

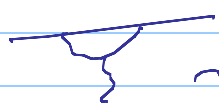
We have started this: grav scalar

$$\Delta \mathcal{L} = \frac{1}{16\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{120}R_{\mu\nu}R^{\mu\nu}\right\} + \frac{1}{16\pi^2}\frac{1}{180}\left(3R_{\mu\nu}R^{\mu\nu} - R^2\right)$$

renormalize

$$C_1^{\overline{MS}} = C_1 + \frac{1}{16\pi^2}\left\{\frac{1}{120} - \frac{7}{60}\right\}\left[\frac{1}{\epsilon} + \ln 4\pi - \ \right]$$

$$C_2^{\overline{MS}} =$$

 $\sim d_1, d_2$

6) Match or <u>measure</u>

$c_i < 10^{65}$

But so far no predictions
- renormalization is needed, but comes from UV
- c_i are not predictions of the EFT

Recall: last time

could be predictions

diff

$8\ell_1 + 2\ell_2 = \frac{3}{192\pi^2}$

σ model

$$\mathcal{M}_{eff} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4}$$

$$+ \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u) + u(u-s)]/v^4$$

$$- \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right]$$

not predictions of EFT

predictions

$\ell_1 = \frac{1}{8m_2^2} + \dots$