Hawking radiation and ultraviolet regulators

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(Received 30 October 1995; revised manuscript received 10 January 1996)

Polchinski has argued that the prediction of Hawking radiation must be independent of the details of unknown high-energy physics because the calculation may be performed using ''nice slices,'' for which the adiabatic theorem may be used. If this is so, then any calculation using a manifestly covariant—and so slice-independent—ultraviolet regularization must reproduce the standard Hawking result. We investigate the dependence of the Hawking radiation on such a short-distance regulator by calculating it using a Pauli-Villars regularization scheme. We find that the regulator scale Λ only contributes to the Hawking flux by an amount that is exponentially small in the large variable $\Lambda/T_H \gg 1$, where T_H is the Hawking temperature, in agreement with Polchinski's arguments. Using the techniques of effective Lagrangians, we demonstrate the robustness of our results. We also solve a technical puzzle concerning the relation between the short-distance singularities of the propagator and the Hawking effect. [S0556-2821(96)03610-7]

PACS number(s): 04.70.Dy, 04.62.+v, 11.10.Gh

I. INTRODUCTION

The prediction that very massive stars must end their days as black holes has by now become deeply ingrained into common astrophysical lore. Our belief in this result rests in no small part on the continued success with which general relativity accounts for observations, both within the solar system and beyond.

Part of the progress of the last 20 years has been the integration of this success into the broader body of laws which describe the other known, nongravitational, interactions. It is now understood that, in spite of the notorious obstacles to constructing a full quantum theory of gravity, semiclassical general relativity can be interpreted as a controllable low-energy approximation to whatever unknown physics might ultimately describe nature on the very shortest of length scales. In this sense, general relativity joins other venerable, but nonrenormalizable, low-energy effective theories [1], and semiclassical calculations are justified for observables that vary on distance scales that are long compared to the Planck length.

Perhaps the biggest surprise to emerge from the study of semiclassical quantum physics in the presence of macroscopic gravitational fields is Hawking's discovery [2], that black holes constantly radiate subatomic particles. These particles dominantly emerge far from the hole with energies that are of order the Hawking temperature: $E \simeq T_H \equiv (4\pi r_s)^{-1}$, where[1] $r_s = 2GM$ is the Schwarzschild radius for a black hole of mass M. Provided that the hole is sufficiently massive, $GM^2 \gg 1$ (or, in cgs units, $M \gg 22\mu$g), the radiation is a long-wavelength effect, and so one expects its semiclassical description to be justified.

It therefore comes as something of a surprise, as was originally emphasized in [3], and more recently in [4,5], to find that the standard derivations of the Hawking effect (in four dimensions) make reference in one way or another to physics at extremely short distances. This is true both of Hawking's original derivation, as well as of more modern alternatives [6].

The short distances arise because the Hawking radiation is defined to be the flux which emerges at very late times, well after all of the transients associated with the stellar collapse itself have passed. However, in the usual derivations the radiation which emerges at the Hawking temperature at such late times is strongly redshifted as it climbs out of the black hole's gravitational well. Alternatively, in the formalism set up in Ref. [6], the outgoing flux is derived from the short-distance form for the radiated particle's two-point (Hadamard) function (see below for details) as its position arguments, x and x', approach one another and the event horizon.

Polchinski [7], on the other hand, has argued persuasively that, in spite of these appearances, Hawking radiation is nevertheless a robust feature of the long-distance theory. His arguments use the ability, in principle, to perform one's calculations using only ''nice slices'' for which curvatures are everywhere small, and for which the adiabatic theorem ensures all high-frequency modes must be in their ground state.

Our purpose here is to present evidence supporting Polchinski's arguments using an explicit calculation of the Hawking radiation in a simple model. We perform the following consistency check on these arguments: if the existence of nice slices guarantees that the Hawking flux is independent of the details of short-distance physics, then any reasonable manifestly covariant, and so slice-independent, ultraviolet regularization must also not affect this flux. We test this by computing the Hawking radiation using a minimally coupled massive scalar field in the presence of a Schwarzschild black hole, using a Pauli-Villars ultraviolet regularization. We are able to implement this regularization by suitably adapting the methods of Fredenhagen and Haag, Ref. [6]. We find that all of the cutoff dependence vanishes exponentially in the limit $\Lambda \gg T_H$, in agreement with Polchinski's arguments.

The difficulty with any calculation which refers to an explicit type of short-distance physics is that one is left wondering to what extent the conclusions drawn depend in a

[1]We use fundamental units, $\hbar = c = k_B = 1$, throughout.

detailed way on its specific form. We address this potential criticism in Sec. III, by showing that the influence of *any* new physics that is local and generally covariant must be smaller than any power of T_H/Λ. We do so by showing that no possible counterterm exists which can contribute to the Hawking flux as seen by observers at late times far from the hole.

The details of this calculation are described in Secs. II and III. In Sec. IV, we resolve an apparent paradox concerning the relation between the Hawking radiation and the absence of short-distance singularities of the two-point function in the regulated theory. Our conclusions are summarized in Sec. V.

II. A REGULATED EXAMPLE

In this section we compute the dependence of the Hawking flux on the short-distance regulator.

We take as our observable the outgoing energy flux per unit time, $\mathscr{F} \equiv -\langle T^r_t \rangle$, as seen at very late times and at a very large distance from the black hole, with the average taken in the state which corresponds to the vacuum at very early times before the black hole has formed. t and r here represent the usual Schwarzschild coordinates, in terms of which $ds^2 = -(1-r_s/r)dt^2 + (1-r_s/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$. $\mathscr{F}$ is related to the total black hole luminosity by $L_H = \int \mathscr{F} r^2 \sin\theta d\theta d\phi$.

For a minimally coupled scalar field the stress tensor is quadratic in the field operator, and so its expectation may be expressed in terms of the coincidence limit of the Hadamard two-point function: $G(x,x') \equiv 1/2\langle \varphi(x)\varphi(x') + \varphi(x')\varphi(x) \rangle$. In our case,

$$\begin{aligned}\mathscr{F} &\equiv -\langle T^r_t \rangle = -\langle T_{tr^\star} \rangle \\ &= -\frac{1}{2}\lim_{x' \to x}\left(\frac{\partial}{\partial t'}\frac{\partial}{\partial r^\star} + \frac{\partial}{\partial r^{\star\prime}}\frac{\partial}{\partial t}\right)G(x,x'), \qquad (2.1)\end{aligned}$$

where $r^\star$ is the ''tortoise'' coordinate: $r^\star \equiv r + r_s \ln[(r/r_s) - 1]$. The problem reduces to the calculation of $G(x,x')$.

A. The regulator

Since $G(x,x')$ is singular as $x' \to x$, the components of the stress tensor usually diverge, and so must be regularized and renormalized. (Off-diagonal components are typically finite in Schwarzschild, however.) We choose to perform this Pauli-Villars regularization, i.e., by introducing additional fields, $\varphi_i(x)$, some with the ''wrong'' sign kinetic energies, in such a way as to ensure the finiteness of the coincidence limit of

$$G_{\rm reg}(x,x') = \sum_i \epsilon_i G_i(x,x'), \qquad (2.2)$$

where $\epsilon_i = \pm$ keeps track of the sign of the corresponding field's kinetic energy. We should also point out that the sum in Eq. (2.2) includes the contribution from the physical field of mass m whose $\epsilon = +$. Our purpose is ultimately to determine how $\mathscr{F}$ depends on the masses of the regularization fields, M_i, in the limit that $M_i \sim \Lambda \gg T_H \gtrsim m$, where Λ is the inverse of a covariantly defined cutoff length (see below), and m is the mass of the original scalar field.

The properties of the regulator fields that are required may be directly calculated from the known divergence structure for minimally coupled free scalar fields propagating through macroscopic background fields. For a scalar field of mass, m, there are three independent types of divergences, which are known to be proportional to the three coefficients[2] [8]:

$$C_0 \equiv m^4[a_0] = m^4,$$

$$C_1 \equiv m^2[a_1] = -\frac{m^2}{6}R,$$

and

$$\begin{aligned}C_2 \equiv [a_2] &= \frac{1}{180}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - \frac{1}{180}R_{\mu\nu}R^{\mu\nu} \\ &\quad -\frac{1}{30}\Box R + \frac{1}{72}R^2. \qquad (2.3)\end{aligned}$$

Cancellation of all of these short-distance singularities among the Pauli-Villars fields is therefore equivalent to the conditions

$$1 + \sum_i \epsilon_i = 0,$$

$$m^2 + \sum_i \epsilon_i M_i^2 = 0, \qquad (2.4)$$

$$m^4 + \sum_i \epsilon_i M_i^4 = 0.$$

As is easily verified, a solution to these equations is given by $\epsilon_1 = \epsilon_2 = +$, $M_1^2 = M_2^2 = 3\Lambda^2 + m^2$; $\epsilon_3 = \epsilon_4 = -$, $M_3^2 = M_4^2 = \Lambda^2 + m^2$; and $\epsilon_5 = -$, $M_5^2 = 4\Lambda^2 + m^2$.

B. Computing the Hawking flux

We may now use $G_{\rm reg}(x,x')$ to compute the Λ dependence of the Hawking flux. We do so by adapting the arguments of Ref. [6] to our example, since this formulation of the calculation is easily applied to massive fields.

Starting from the definition of $G_{\rm reg}(x,x')$, and Eq. (2.1), we see that the Hawking flux may be simply written as $\mathscr{F} = \Sigma_i \epsilon_i \mathscr{F}_i$, where $\mathscr{F}_i$ is the Hawking flux due to a minimally coupled scalar field of mass M_i. A straightforward application of the techniques of Ref. [6] gives the usual result

[2]These expressions use the conventions of Ref. [9].

$$\mathcal{F}_i(M_i)=\frac{1}{4\pi^2r^2}\sum_{\ell m}|Y_{\ell m}(\theta,\phi)|^2 \times\int_{M_i}^{\infty}d\omega|\mathcal{T}_{\ell}(\omega,M_i)|^2\frac{\omega}{e^{\omega/T_H}-1}. \quad (2.5)$$

In this expression, (r,θ,ϕ) are the Schwarzschild coordinates for the point at which $\mathcal{F}_i$ is computed, and $Y_{\ell m}$ are the usual spherical harmonics. $|\mathcal{T}_{\ell}(\omega,M_i)|^2\leqslant 1$ is the probability that an outgoing particle of mass M_i and energy ω (as seen by the stationary observers at infinity) is transmitted from the event horizon ($r^{\star}\to-\infty$) out to infinity, rather than being scattered back to the horizon by the black hole's gravitational field.

Since the regulator fields all satisfy $M_i\gg T_H$, it suffices to use the asymptotic form for the flux in this limit. In this limit the frequency integral may be bounded from above:

$$\int_{M_i}^{\infty}d\omega|\mathcal{T}_{\ell}(\omega,M_i)|^2\frac{\omega}{e^{\omega/T_H}-1}\leqslant 2\int_{M_i}^{\infty}d\omega\,\omega e^{-\omega/T_H} \leqslant 2T_H^2\left(1+\frac{M_i}{T_H}\right)e^{-M_i/T_H}. \quad (2.6)$$

We see that, for $\mathcal{F}_i$, every term in the sum over ℓ is exponentially small in Λ/T_H. Of course, this is just what would be expected for a thermal radiation spectrum.

One might worry that, although each term in the sum over ℓ is exponentially small, it may be that the series sums to a result which is *not* exponentially suppressed. This does not happen, however, because a much stronger bound is possible for $|\mathcal{T}_{\ell}|^2$ when ℓ becomes sufficiently large. The better bound arises because for large angular momenta the transmission probability, $|\mathcal{T}_{\ell}(\omega,M_i)|^2$ goes to zero. This can most easily be seen by recasting the scattering problem in terms of the quantum mechanics of a single particle moving in the presence of an effective ''potential'':

$$V_{\rm eff}\equiv M_i^2\left(1-\frac{r_s}{r}\right)\left(1+\frac{\ell(\ell+1)}{M_i^2r^2}+\frac{r_s}{M_i^2r^3}\right) \approx M_i^2\left(1-\frac{r_s}{r}\right)\left(1+\frac{\ell^2}{M_i^2r^2}\right). \quad (2.7)$$

This last, approximate, form has been simplified using $\ell\gg 1$ and $M_ir_s\gg 1$. Classical evolution in this potential simply predicts $|\mathcal{T}_{\ell}|^2=1$ when ω lies above the potential for all r, and $|\mathcal{T}_{\ell}|^2=0$ when ω is below the barrier for some r. For the above potential, however, there is no barrier at all to escape for $\ell\leqslant L\equiv\sqrt{3}M_ir_s$, since only for these ℓ's can the centrifugal contribution dominate the gravitational attraction. For $\ell>L$, on the other hand, $V_{\rm eff}$ has a maximum for $r=r_{\rm max}\gtrsim r_s$ that can reflect a potentially outgoing particle, and so transmission is forbidden for $\omega<V_{\rm max}$. But since the height of the barrier, $V_{\rm max}\sim\ell/M_ir_s$, grows for large ℓ, reflection eventually becomes inevitable for sufficiently large ℓ. Physically, particles with large ℓ, but fixed ω, are not sufficiently radially directed to escape to infinity once they try to climb out of the black hole's gravitational well. As a result the sum over ℓ that appears in $\mathcal{F}_i$ is eventually cut off for sufficiently large ℓ.

We conclude, then, that at least for this regularization, the contribution of very-short-distance physics, at distances $\sim 1/\Lambda$, to the Hawking flux is exponentially suppressed by the large ratio Λ/T_H. This result raises two further questions.

(1) To what extent does this result depend on the details of the small-distance physics? Could other regularizations lead to different conclusions, such as to corrections proportional to Λ^{-p}, for some p? We claim not, and demonstrate this robustness in Sec. III.

(2) A more technical question is: since the technique of Ref. [6] relates the Hawking flux to the singularity of the two-point function, $G(x,x')$ in the coincidence limit at the horizon, how can a nonzero flux have been obtained using a regularized propagator such as the function $G_{\rm reg}(x,x')$ used here? We deal with this question in Sec. IV.

III. THE GENERAL ARGUMENT

In this section we wish to argue for the robustness of the last section's result. We do so by demonstrating that *any* other physically acceptable regularization can also only give an exponentially small contribution to the Hawking flux.

Our starting point is the following expression for the lowest-order (i.e., one-loop) vacuum expectation value of the stress tensor [10]

$$\langle T^{\mu\nu}\rangle=\left[\frac{2}{\sqrt{-g}}\frac{\delta\Gamma}{\delta g_{\mu\nu}}\right]_{g_s,\varphi_s}, \quad (3.1)$$

where the subscripts indicate that the result is to be evaluated at the background field configuration, $(g_s)_{\mu\nu}$ and φ_s. In the present instance we take these to be the classical solutions: a Schwarzschild metric with a constant scalar field which minimizes the scalar potential. The functional $\Gamma[g,\varphi]$ in the above expression represents the generator for the one particle irreducible (1PI) Green's functions for the scalar field, φ, in the presence of a background metric, $g_{\mu\nu}$.

We know how physics at very-short-distance scales, $\lambda\sim 1/\Lambda$, can contribute to $\langle T_{\mu\nu}\rangle$ once we determine how it can appear in $\Gamma[g,\varphi]$. But here we may take advantage of the basic property that makes effective Lagrangian techniques so powerful elsewhere in physics: the influence of short-distance physics on long-wavelength observables may be parametrized in terms of a set of real, local and generally covariant operators which involve only the long-wavelength degrees of freedom. That is,

$$\Gamma[g,\varphi]=S_{\rm cl}+\Gamma_{\rm loop}[g,\varphi]+\Gamma_{\Lambda}[g,\varphi], \quad (3.2)$$

where $S_{\rm cl}$ is the classical action, $\Gamma_{\rm loop}[g,\varphi]$ represents the long-distance loop contributions (as are usually computed), and the ''counterterms'' in $\Gamma_{\Lambda}[g,\varphi]$ are the result of short-distance loops from frequencies greater than Λ. This last contribution cannot be evaluated in detail without knowing what physics lies beyond Λ, but in general must take the form

$$\Gamma_\Lambda[g,\varphi]=\sum_n c_n\Lambda^{4-d_n}\int d^4x O_n(\varphi,g_{\mu\nu}). \quad (3.3)$$

Here n runs over all possible local operators, $O_n(\varphi,g)$, which we take to be organized with increasing dimension: $(\text{mass})^{d_n}$. In all of these expressions the fields $g_{\mu\nu}$ and φ can vary appreciably only over distances that are long compared to λ. This is because it is only in this limit that the short-distance physics can be represented by local operators.

The main point is that *any* type of physically acceptable short-distance physics may be parametrized in terms of this type of real, local, and generally covariant effective Lagrangian. Differing types of short-distance physics that share the same low-energy particle content, and respect the same low-energy symmetries, can only differ in their predictions for the values of the various coefficients, c_n. It is important to notice that we do *not* restrict ourselves here to those terms in $\Gamma_\Lambda[g,\varphi]$ which diverge as $\Lambda\to\infty$.

In particular, in order for *any* short-distance modification of the minimally coupled scalar model of the previous section to contribute differently to the Hawking radiation, it must be possible to represent this difference in terms of a real, local, generally covariant contribution to L_{eff}. A contribution that is suppressed only by k powers of Λ^{-1} (as opposed to being exponentially small) should show up as an effective operator having dimension $(\text{mass})^{k+4}$. It only remains to show that no such operator exists that is a generally covariant, local polynomial of the fields, ϕ, $g_{\mu\nu}$ and their derivatives. As a result, new physics at scale Λ can only contribute an amount that is smaller than any power of Λ^{-1}.

An effective operator in $\Gamma_\Lambda[g,\varphi]$ can only contribute to the Hawking radiation if its contribution, $\delta\langle T_{\mu\nu}\rangle$, to the vacuum expectation value of the stress tensor satisfies the following two properties: (1) In order to contribute to $\mathscr{F}$, its off-diagonal, $\delta\langle T_{tr^\star}\rangle$, component must not be zero; and (2) this off-diagonal component must fall off like $1/r^2$ as $r\to\infty$ in order to contribute a nonvanishing amount to the particle flux, L_H, at large distances from the black hole.

The argument that rules out any such term is very simple to give. Any such term must be a symmetric, conserved tensor that is constructed from local powers of the fields and their derivatives. Since the background scalar-field configuration, φ_s, is constant, it only contributes in a trivial way. The only other fields are the metric, and its derivatives: curvature tensors, their covariant derivatives, etc.

The only possible term which does not involve the curvature tensor would be $\delta\langle T_{\mu\nu}\rangle\propto(g_s)_{\mu\nu}$, for which the t–$r^\star$ element is zero. But all components of the curvature tensor and its covariant derivatives fall off for large r at least as fast as $1/r^3$, and so approach zero too quickly to contribute a flux at infinity.

We see in this way that, with the given particle content, the contribution of short-distance physics is always smaller than any power of Λ^{-1}.

IV. HAWKING RADIATION AND THE ABSENCE OF SINGULARITIES

The result of the last section raises another question. We have computed the Hawking radiation in a regulated theory having a completely smooth two-point function. But it is also straightforward to show, by trivially extending the arguments of Ref. [6] to massive fields, that a coincidence limit of the form $G(x,x')\sim 1/[4\pi^2\sigma(x,x')]+(\text{less singular})$, where $\sigma(x,x')$ denotes the proper separation between the points x and x', is required near $r=r_s$ in order to produce the Hawking radiation. That is, in the approach of Ref. [6] the Hawking flux is completely determined by the coefficient of this $1/\sigma$ singularity of the two-point function, $G(x,x')$, when the coincidence limit is taken near the black hole event horizon. The question therefore is: How can a nonzero flux be obtained using a regularized propagator which is smooth in the coincidence limit? The present section is devoted to the resolution of this apparent contradiction.

The starting point for the analysis of Ref. [6] is the observation that Eq. (2.1) allows us to compute the Hawking flux at a point $(T,\mathscr{R},\Theta,\Phi)$, at large distances from the black hole and at late times, given knowledge of $G(x,x')$ in the neighborhood of this point. In Ref. [6] the two-point function at large distances from the black hole, $G(X_1,X_2)$, is related to its values on an earlier spacelike hypersurface using the surface independence of the Klein-Gordon inner product:

$$(f,g)=\int_\Sigma f^*\overleftrightarrow{\partial}\mu g\, d\Sigma^\mu \quad (4.1)$$

provided that the functions f and g satisfy the Klein-Gordon equation. This leads to the expression

$$G(X_1,X_2)=\int_{\Sigma_\tau}\int_{\Sigma_\tau} d\Sigma^1_\mu d\Sigma^2_\nu G(x_1,x_2) \times\overleftrightarrow{\partial}\partial_{1\mu}\overleftrightarrow{\partial}_{2\nu}f(x_1)f^*(x_2), \quad (4.2)$$

where both integrals are taken over the same timelike surface, Σ_τ which we may take to be a surface of constant $\tau=t+r^\star-r$. The measure for such a surface is $d\Sigma^\mu=n^\mu r^2\sin^2\theta dr d\theta d\phi$, with n^μ the unit normal to Σ_τ. Explicitly,

$$n\cdot\partial=\left(1+\frac{r_s}{r}\right)\frac{\partial}{\partial\tau}-\frac{r_s}{r}\frac{\partial}{\partial r}. \quad (4.3)$$

The function $f(x)$ which appears in Eq. (4.2) is the particular solution to the Klein-Gordon equation which satisfies the following "initial" conditions, which we choose to specify on a late-time constant-t surface which contains the point $X=(T,\mathscr{R},\Theta,\Phi)$ at which the Hawking flux is to be measured:

$$f(x)\Big|_{t=T}=0,$$
$$\partial_t f(x)\Big|_{t=T}=\delta^3(\vec{x}-\vec{X}). \quad (4.4)$$

The vector symbol here denotes the three coordinates which specify a point on the surface $t=T$.

The Fredenhagen-Haag derivation [6] crucially relies on this surface independence of the Klein-Gordon inner product, $(f,g)=\int_\Sigma f^* \overleftrightarrow{\partial}_\mu g\, d\Sigma^\mu$, when the functions f and g satisfy the Klein-Gordon equation. In Eq. (4.2), this is applied in particular to the two-point function, $G(x,x')$. The resolution of the apparent paradox therefore relies on the fact that a regulated propagator like $G_{\rm reg}(x,x')$ does *not* satisfy the Klein-Gordon equation, but rather satisfies a more complicated higher-derivative equation of motion. The conserved inner product for this equation of motion also involves higher-derivative corrections, and these corrections are what generate the Hawking flux from a nonsingular two-point function.

We next illustrate this argument with an explicit calculation. Rather than dealing with the cumbersome details of the five regulator fields that are used in the text, for clarity of presentation we instead present an example which uses just one regulator field. Consider, therefore, the following two-point function:

$$\hat{G}(x,x') \equiv G_{m^2}(x,x') - G_{M^2}(x,x'), \tag{4.5}$$

where $G_{m^2}(x,x')$ and $G_{M^2}(x,x')$, respectively, denote the two-point functions for free scalar fields of mass m and $M \gg m$. Comparing to the short-distance expansion of Ref. [8] shows that the coincidence limit of $\hat{G}(x,x')$ is at worst $\sim \ln\sigma(x,x')$, for Schwarzschild spacetime. Even though this is less singular than $1/\sigma(x,x')$, our goal here is to show that $\hat{G}(x,x')$ nevertheless produces a nonzero Hawking flux.

In order to apply the methods of Ref. [6], we must first find what equation of motion $\hat{G}(x,x')$ satisfies, and then construct the corresponding conserved "inner product" for this equation. As is simple to check, the equation of motion is

$$\frac{1}{M^2-m^2}(\Box - m^2)(\Box - M^2)\hat{G}(x,x') = 0. \tag{4.6}$$

The conserved "inner product" for two solutions, f and g, of this equation then is

$$[f,g] = -\frac{M_+^2}{M_-^2}\int_\Sigma d\Sigma^\mu f^* \overleftrightarrow{\partial}_\mu g + \frac{1}{M_-^2}\int_\Sigma d\Sigma^\mu f^* \overleftrightarrow{\partial}_\mu \Box g + \frac{1}{M_-^2}\int_\Sigma d\Sigma^\mu (\Box f^*) \overleftrightarrow{\partial}_\mu g, \tag{4.7}$$

where $M_\pm^2 \equiv M^2 \pm m^2$, and Σ is a spacelike surface. Clearly this expression approaches the usual Klein-Gordon one in the limit $M \to \infty$.

Using this expression to write the analogue of Eq. (4.2) gives the result

$$\hat{G}(X_1,X_2) = \sum_{j=1}^{4} \hat{G}_j(X_1,X_2), \tag{4.8}$$

where

$$\hat{G}_1(X_1,X_2) = \frac{1}{M_-^4}\int_\Sigma\int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu \hat{G}(x_1,x_2) \times \overleftrightarrow{\partial}_{1\mu}\overleftrightarrow{\partial}_{2\nu} F(x_1)F^*(x_2),$$

$$\hat{G}_2(X_1,X_2) = \frac{1}{M_-^4}\int_\Sigma\int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [\Box_1 \hat{G}(x_1,x_2)] \times \overleftrightarrow{\partial}_{1\mu}\overleftrightarrow{\partial}_{2\nu} f(x_1)F^*(x_2), \tag{4.9}$$

$$\hat{G}_3(X_1,X_2) = \frac{1}{M_-^4}\int_\Sigma\int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [\Box_2 \hat{G}(x_1,x_2)] \times \overleftrightarrow{\partial}_{1\mu}\overleftrightarrow{\partial}_{2\nu} F(x_1)f^*(x_2),$$

$$\hat{G}_4(X_1,X_2) = \frac{1}{M_-^4}\int_\Sigma\int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [\Box_1\Box_2 \hat{G}(x_1,x_2)] \times \overleftrightarrow{\partial}_{1\mu}\overleftrightarrow{\partial}_{2\nu} f(x_1)f^*(x_2).$$

In these expressions $F(x) \equiv \Box f(x) - M_+^2 f(x)$. The function $f(x)$ must also satisfy the "initial conditions"

$$f(x)|_{t=T} = 0,$$
$$\partial_t f(x)|_{t=T} = 0,$$
$$\Box f(x)|_{t=T} = 0, \tag{4.10}$$
$$\partial_t \Box f(x)|_{t=T} = M_-^2 \delta^3(x),$$

which play the role here of Eq. (4.4) in the Klein-Gordon case.

As fearsome as it looks, this initial-value problem can be solved, and leads to functions, $f^-(x)$, which are basically identical with those that are found for the Klein-Gordon case. In particular, their support becomes infinitely small as $(T-t) \to \infty$, requiring a coefficient function that varies like $1/\sigma(x_1,x_2)$ near the horizon. The new feature, though, is that the function that must be this singular involves not just $\hat{G}(x_1,x_2)$, but also its *derivatives*. It is these derivative terms that save the day: acting on $\hat{G}(x,x')$ they convert its $\ln\sigma(x,x')$ behavior into the $1/\sigma(x,x')$ that is required for a nonzero result.

V. SUMMARY

We have presented a derivation of the Hawking radiation within a simple model for which the ultraviolet regularization has been made explicit. This calculation permits the regularization dependence of the Hawking flux to be explicitly displayed. It is found that the cutoff dependence is exponentially small in the limit that $\Lambda/T_H \gg 1$. Since the Pauli-Villars regularization used is slice independent, this result agrees with what one would expect from the "nice-slice" argument in favor of the irrelevance of the details of high-energy physics on the prediction of Hawking radiation.

We show in Sec. III that the conclusion that the contributions of the details of the short-distance physics to the Hawking flux are exponentially suppressed in the ratio $\Lambda/T_H \gg 1$ is not specific to the Pauli-Villars regularization but is a generic feature of any generally covariant short-distance regularization (modification) of short-distance physics. Using the effective Lagrangian techniques, we rule out the existence of any covariant, local cutoff dependent counterterm which can contribute to the Hawking flux at late times far from the hole.

The computation scheme of Fredenhagen and Haag [6] is used throughout, in which the Hawking radiation is directly related to the coincident singularity of the two-point function as both of its position arguments approach one another and the event horizon. We show in Sec. IV that there is no contradiction in this approach between having a nonzero Hawking flux in the regulated theory, even though the resulting regulated two-point function is nonsingular in the coincidence limit.

ACKNOWLEDGMENTS

We thank Ted Jacobson for explaining to us the mysteries of Ref. [6], and both Ted and Rob Myers for useful discussions. This research was partially funded by funds from the NSERC of Canada and les Fonds FCAR du Québec.

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