Interactions 4

Note Title 2/25/2010

Feynman rules $\mathcal{L}_I = -\frac{\lambda}{4}\phi^4$

$$T_{fi} = (2\pi)^4 \frac{\delta^4(p_i - p_f)}{\sqrt{2\omega_1 \cdots}} \left(-i\mathcal{M}\right)$$

$\longleftarrow$ Rules for $-i\mathcal{M}$

1) Draw all possible connected diagrams (to some order in λ) *

2) Reserve to later "self energy" diagrams

3) For any diagram, $-i\mathcal{M} =$

a) factor of $\langle -iH \rangle = \langle +i\mathcal{L} \rangle = -i\frac{\lambda}{4} \times 4! = -i6\lambda$

drop $\frac{1}{\sqrt{2\omega}} e^{ip\cdot x}$

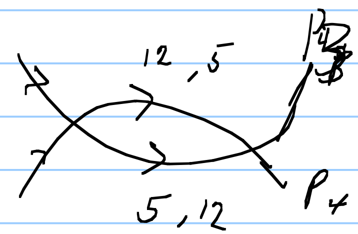
b) momentum cons. at each vertex

c) any internal line $\quad \frac{i}{q^2 - m^2}$

d) Any momentum not fixed $\Rightarrow \int \frac{d^4 q}{(2\pi)^4}$ ✓

e) Two bosons of same type in loop $\Rightarrow \frac{1}{2}$ ✓

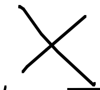
ie $\frac{1}{2}$ in $\quad 10 = p_1$ $\quad 7 = p_2$



same state

$\frac{1}{2}$ removes double counting

Example $\lambda\phi^4$ to $\mathcal{O}(\lambda^2)$



$\mathcal{O}(\lambda)$ $\boxed{-iM = -6i\lambda}$

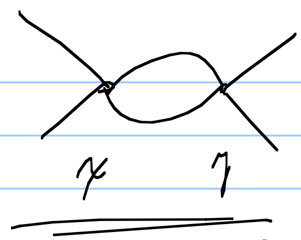
$$S = U_I(\infty, -\infty) = 1 - i\frac{\lambda}{4}\int d^4x\,\phi^4 - \frac{1}{2}T\int d^4x\,d^4y\left[\frac{\lambda}{4}\phi^4(x)\,\frac{\lambda}{4}\phi^4(y)\right] + \cdots$$

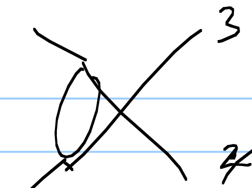
$\mathcal{O}(\lambda^2)$

$$T_{fi}^{(2)} = \langle p_3, p_4|\, -\frac{T}{2}\int d^4x\,d^4y\left(\frac{\lambda}{4}\phi^4(x)\;\phi^4(y)\right)|p_1\; p_2\rangle$$

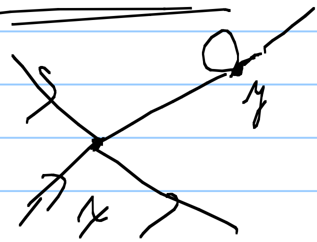
Wick's Thm any ϕ that dont act on external state $\langle 0|T(\phi(x)\,\phi(y))|0\rangle$

$$T(\phi^4) = \cdots\cdots\; :\phi\,\phi\,\phi\,\phi:\; \underline{\underline{\langle 0|T(\phi\,\phi)|0\rangle}}\,\langle 0|T\phi\phi|0\rangle$$

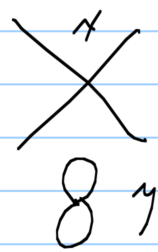




✓



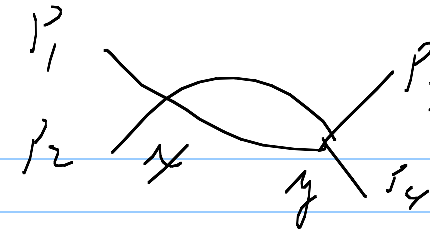
"Self energy"



"Disconnected"

drop

Calculation



$$-\frac{1}{2}\left(\frac{\lambda}{4}\right)^2 \langle p_3\, p_4 |\, T(\phi^4(x)\, \phi^4(y))\, | p_1\, p_2 \rangle$$

Counting factor

2 ways to act on initial state $\quad x$ or $y \qquad$ (choose x)

4 ways to annih. p_1 from $\phi^4(x)$

3 " " p_2

4 ways to create p_3 from $\phi^4(y)$

3 " " p_4

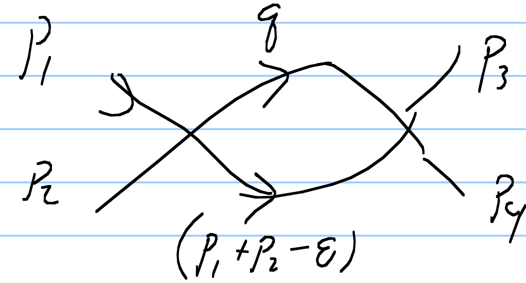
– left with $\langle 0 |\, T\, \phi(x)\phi(x)\phi(y)\phi(y)\, | 0 \rangle$

2 way to connect them $\Rightarrow 2 \langle 0 | T(\phi(x)\, \phi(y)) | 0 \rangle \langle 0 | T(\phi(x)\, \phi(y)) | 0 \rangle$

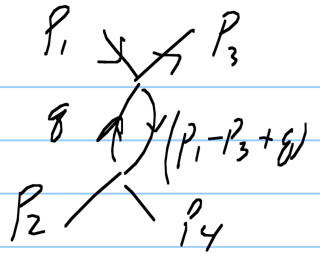
$$T = \int d^4x\, d^4y\; \frac{1}{2}\left(\frac{\lambda}{4}\right)^2 (4\times 3\times 2)^2\, \frac{e^{-i(p_1+p_2)\cdot x}}{\sqrt{2\omega_1 2\omega_2}}\, \frac{e^{+i(p_3+p_4)\cdot y}}{\sqrt{2\omega_3 2\omega_4}} \int \frac{d^4q}{(2\pi)^4}\, \frac{i\, e^{+iq\cdot(x-y)}}{q^2-m^2+i\epsilon} \int \frac{d^4q'}{(2\pi)^4}\, \frac{i\, e^{+iq'\cdot(x-y)}}{q'^2-m^2+i\epsilon}$$

$$= \int \frac{d^4q}{(2\pi)^4}\, \frac{d^4q'}{(2\pi)^4}\, \frac{(2\pi)^4\delta(p_1+p_2-q-q')\,(2\pi)^4\delta^4(p_3+p_4-q-q')}{\sqrt{2\omega_1 \cdots 2\omega_4}}\; -\frac{1}{2}(6\lambda)^2\, \frac{i}{q^2-m^2+i\epsilon}\, \frac{i}{q'^2-m^2+i\epsilon}$$

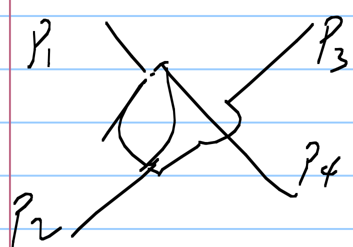
$$= \frac{(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)}{\sqrt{2\omega_1 \cdots 2\omega_4}}\, (-i\mathcal{M})$$



$$-i\mathcal{M} = \frac{1}{2}(-6i\lambda)^2\; \underbrace{\int \frac{d^4q}{(2\pi)^4}\, \frac{i}{q^2-m^2+i\epsilon}\, \frac{i}{(p_1+p_2-q)^2-m^2+i\epsilon}}_{I(p_1+p_2)}$$



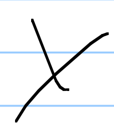
$$-i\mathcal{M} = \frac{1}{2}(-6i\lambda)^2\, I(p_1-p_3)$$



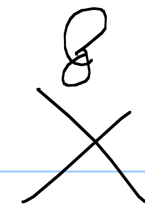
$$-i\mathcal{M} = \frac{1}{2}(-6i\lambda)^2\, I(p_1-p_4)$$

To $O(\lambda^2)$

$$-i\mathcal{M} = -6i\lambda + \frac{1}{2}(-6i\lambda)^2\left[I(p_1+p_2) + I(p_1-p_3) + I(p_1-p_4)\right]$$

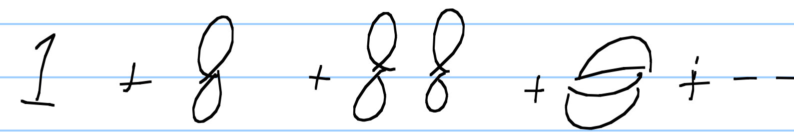


Dropping disconnected diagrams



Look at amplitude to vacuum

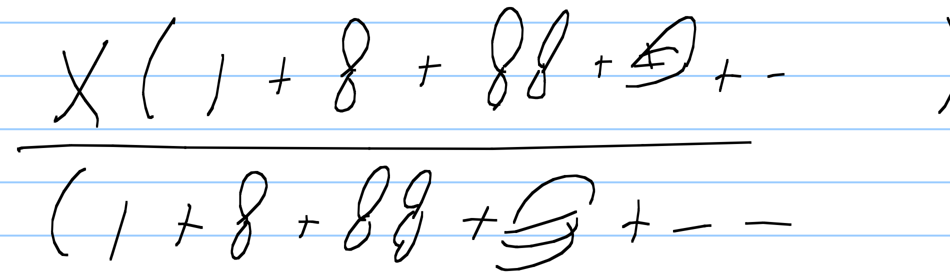
$$\langle 0 | U_I(\infty, -\infty) | 0 \rangle =$$



$= $ at most a phase $e^{i\theta}$

Better def of scattering

$$T_{fi} = \frac{\langle f | U(\infty, -\infty) | i \rangle}{\langle 0 | U(\infty, -\infty) | 0 \rangle} =$$



Disconnected diagram are common phase

$\Longrightarrow$ drop

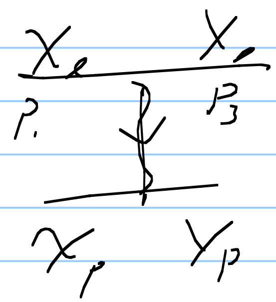
Feynman rules for $\mathcal{L}_I = -g\,\chi^\dagger \chi \phi$

Vertex 

$= \langle -i\mathcal{H}_I \rangle = \langle +i\mathcal{L}_I \rangle = -ig$

↑ drop $\frac{1}{\sqrt{2\omega}}$

all else is same



$$\Rightarrow \quad -i\mathcal{M} = (-ig_e)\,\frac{i}{q^2 - m^2 + i\epsilon}\,(-ig_p)$$

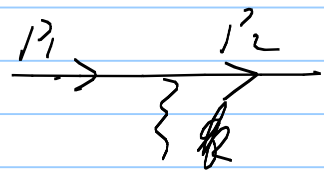
↑ $(p_1 - p_3)^2$

Scalar QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi)$$

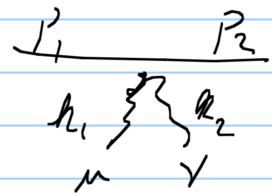
$$\mathcal{L}_I = -qA_\mu\left(\phi^* i \overleftrightarrow{\partial}^\mu \phi\right) + q^2 A_\mu A^\mu \phi^*\phi$$

Vertex



$= -iq(p_1+p_2)^\mu$

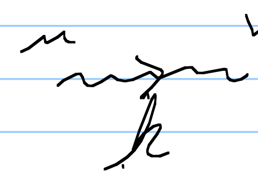
charge



$= 2iq^2 g_{\mu\nu}$

Propagator – photon

$$\frac{-ig^{\mu\nu}}{k^2+i\epsilon}$$



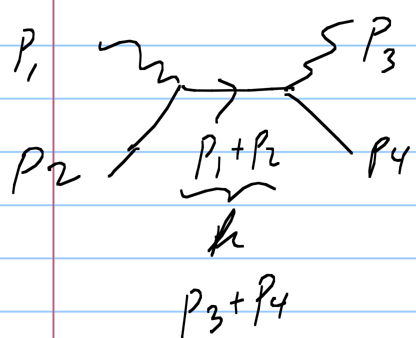
* External photons ϵ_μ or ϵ^*_μ

↑ initial ↑ final

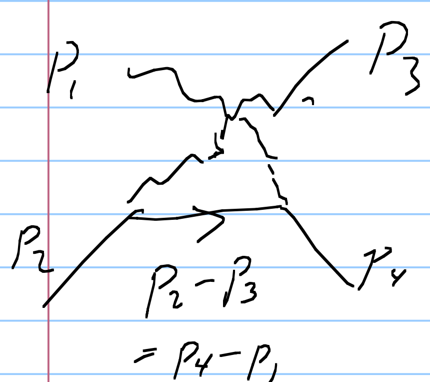
Example

Compton scattering $\gamma + \text{"}e\text{"} \to \gamma + \text{"}e\text{"}$

$$\left(+2i\, g_{\mu\nu}\, g_0^2\right) \epsilon_\mu(p_1)\, \epsilon^*_\nu(p_2)$$



$$\left[-ig\,\underbrace{\left(p_1+(p_1+p_2)\right)^\mu}_{(2p_2+p_1)} \frac{i}{(p_1+p_2)^2-m^2}(-ig)\,\underbrace{\left(p_4+(p_3+p_4)\right)^\nu}_{(2p_4+p_3)}\right]\epsilon_\mu\, \epsilon^*_\nu$$



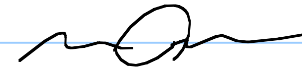
$$\left[(-ig)(2p_2-p_3)^\mu \frac{i}{(p_2-p_3)^2-m^2}(-ig)(2p_4-p_1)^\nu\right]\epsilon_\mu\, \epsilon^*_\nu$$

Fermions + QED

- $-T = (\quad)$ no $\frac{1}{\sqrt{2\omega}}$

- External fermions
  - $u(p)$ incoming f
  - $\bar{u}(p)$ outgoing f
  - $v(p)$ outgoing $\bar{f}$
  - $\bar{v}(p)$ incoming $\bar{f}$

Propagator $\dfrac{i}{\not{p} - m + i\epsilon}$

~~**~~ -1 for closed fermion loop 

QED  $= -iq\gamma_\mu$

General vertex $= \langle -i\mathcal{H}_I \rangle = \langle +i\mathcal{L}_I \rangle$ drop external stuff