Note Title 10/8/2009

Gauge Theory 5 Oct 8, 2009

$(g-2)$

$g = 2$ Dirac theory

$$(i\not{D} - m)\psi = 0 \quad \Longrightarrow \quad (i\not{D} + m)(i\not{D} - m)\psi$$

$$= (-\not{D}\not{D} - m^2)\psi$$

$$[D_\mu, D_\nu] = i e F_{\mu\nu}$$

$$\gamma^\mu \gamma^\nu D_\mu D_\nu = \frac{1}{2}\left[\{\gamma_\mu, \gamma_\nu\} + [\gamma_\mu, \gamma_\nu]\right] D_\mu D_\nu = \underline{D_\mu D^\mu - \frac{e}{2}\sigma^{\mu\nu} F_{\mu\nu}}$$

B field $\quad A_1 = -\frac{1}{2} B y, \quad A_2 = \frac{1}{2} B x \qquad F_{12} = \partial_1 A_2 - \partial_2 A_1 = B$

$$D_i^2 = \partial_i^2 - ie\left(\partial_i A_i + A_i \partial_i\right) + \cancel{A_i^2}$$

$$= \partial_i^2 - 2\,\frac{ieB}{2}\,\underbrace{\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)}_{\vec{r}\times\vec{p}\,=\,L}$$

$$\sigma^{ij} = \varepsilon^{ijk}\begin{pmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{pmatrix} = \varepsilon^{ijk}\, 2\vec{S}_k$$

$$\Rightarrow \sigma^{12}F_{12} + \sigma^{21}F_{21} = 2\alpha\, 2\vec{S}\cdot\vec{B}$$

Then

$$\left[\partial_0^2 + m^2 - \nabla^2 - eB\underbrace{(L + 2S)}_{g=2}{}_z\right]\psi = 0$$

Gordon Decomposition

$$\bar{u}(p')\gamma_\mu u(p) = \bar{u}(p')\left[\frac{\not{p}'\gamma_\mu + \gamma_\mu \not{p}}{2m}\right] u(p)$$

Let $\not{p} = \frac{1}{2}(\not{p}+\not{p}') - \frac{1}{2}(\not{p}'-\not{p})$

$\not{p}' = \quad$ " $\quad + \quad$ " $\quad \Big\}$ first $\Rightarrow$ $\underline{\not{P}}\gamma_\mu + \gamma_\mu \not{P}$

$\not{q}\gamma_\mu - \gamma_\mu \not{q}$

$$\underline{\bar{u}(p')\gamma_\mu u(p)} = \bar{u}(p')\left[\frac{(p+p')^\mu}{2m} + i\frac{\sigma^{\mu\nu}}{2m}q_\nu\right] u(p)$$

convection current like scalar

Spin

$g = 2$

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} = \frac{g}{2} \frac{e}{2m} \vec{\sigma} \cdot \vec{B}$$

$$\vec{S} = \vec{\sigma}/2$$

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

$$\vec{\sigma} \cdot \vec{B} \rightarrow \frac{1}{2} \bar{u}\, \sigma^{\mu\nu} u\, F_{\mu\nu} \rightarrow \frac{1}{2} \bar{u}\, \sigma^{\mu\nu} u\, F_{\mu\nu}$$

$$\mathcal{H}_M = \frac{g}{2} \frac{e}{2m} \bar{\psi}\, \sigma^{\mu\nu} \psi \frac{F_{\mu\nu}}{2} \Leftarrow$$

Matrix elements

$$\langle \mathcal{H}_m \rangle = \frac{g}{2} \frac{e}{2m} \bar{u}\, \sigma^{\mu\nu} u \frac{2 q_\mu}{2} \epsilon_\nu = \bar{u} \frac{g}{2} \frac{e}{2m} \sigma^{\mu\nu} q_\nu\, u\, \epsilon_\nu$$

$\uparrow g = 2$

General Form for Matrix Element

$$\mathcal{M}_\mu = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i}{2m} F_2(q^2)\sigma^{\mu\nu} q_\nu\right] u(p) \qquad \checkmark$$

Lorentz, parity, gauge inv.

$q^\mu = (p'-p)^\mu$

$$\mathcal{M}_\mu = \bar{u}(p')\left[a\gamma_\mu + b(p+p')^\mu + c\,q^\mu + d\,\sigma^{\mu\nu} q_\nu + e\,\sigma^{\mu\nu} P_\nu\right] u(p)$$

Gordon $\to$ $a + e$

$$q^\mu \mathcal{M}_\mu = \bar{u}(p')\left[0 + 0 + c\,q^2 + d\,0 + e\,\sigma^{\mu\nu} q_\mu P_\nu\right] u(p)$$

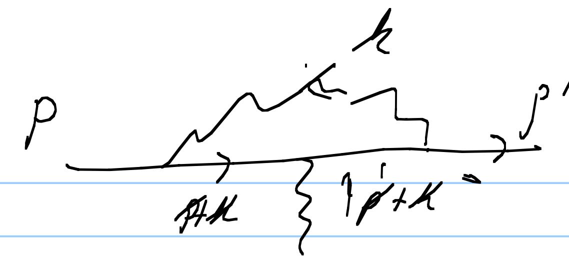
$c = 0$, $d = 0$

$$\mu = \frac{e}{2m}\left[F_1(0) + F_2(0)\right] = \frac{e}{2m}\left[1 + F_2(0)\right]$$

$F_1(0) = 1$

$$(g-2) = \underline{\underline{F_2(0)}}$$

Calculating $g-2$



$$-ie\Gamma_\mu = \int \frac{d^4k}{(2\pi)^4} \quad -ie\gamma_\alpha \quad \frac{i}{\not p' + \not k - m} \quad -ie\gamma_\mu \quad \frac{i}{\not p + \not k - m} \quad -ie\gamma^\alpha \quad \frac{-i}{k^2}$$

$$N^\mu = \gamma_\alpha \, (\not p' + \not k + m)\, \gamma^\mu \, (\not p + \not k + m)\, \gamma^\alpha$$

$$D = k^2 \left[(p+k)^2 - m^2 \right] \left[(p'+k)^2 - m^2 \right]$$

<u>Goal</u> – Use Gordon in reverse

$$\Gamma_\mu = \bar u(p') \left[\gamma_\mu \left[F_1(q^2) + F_2(q^2) \right] - \frac{1}{2m} (p+p')^\mu F_2(q^2) \right] u(p)$$

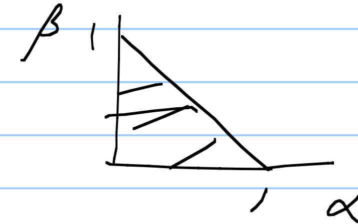
reduce every term to γ_μ or $(p+p')^\mu$

drop

Feynman parameters

$$\frac{1}{abc} = 2\int_0^1\int_0^1\int_0^1 d\alpha\, d\beta\, d\gamma \,\frac{\delta(1-\alpha-\beta-\gamma)}{(a\alpha+\beta b+c\gamma)^3}$$

$$= 2\int\int_{\text{Tri}} d\alpha\, d\beta \,\frac{1}{(\quad)^3}$$



$(\alpha+\beta) < 1 \Rightarrow \int_0^1 d\alpha \int_0^{1-\alpha} d\beta$

$\Rightarrow$ Symmetric

$$\frac{1}{D} = 2\int_{\text{Tri}} d\alpha\, d\beta \,\frac{1}{\mathcal{D}^3}$$

$$\mathcal{D} = \left[\alpha\left((p'+k)^2-m^2\right)+\beta\left((p+k)^2-m^2\right)+\gamma k^2\right]$$

$$(p'+k)^2 - m^2 = k^2 + 2p'\cdot k \qquad p'^2 = m^2$$

$$(p+k)^2 - m^2 = k^2 + 2p\cdot k$$

$$p'^2 = (p+q)^2 \Rightarrow m^2 = m^2 + 2p\cdot q + q^2 \qquad \Rightarrow 2p\cdot q = -q^2$$

$$2p'\cdot q = +q^2$$

$$p\cdot p' = (p+q)\cdot p = m^2 + p\cdot q = m^2 \cancel{+ q^2/2}$$

$\uparrow$ drop

$$\mathcal{D} = [k^2 + 2k\cdot(\alpha p' + \beta p)]$$

Shift $k^\mu + (\alpha p'^\mu + \beta p^\mu) = \ell^\mu$

$$\mathcal{D} = [\ell^2 - (\alpha p' + \beta p)^2] = [\ell^2 - (\alpha^2 m^2 + \beta^2 m^2 + 2\alpha\beta m^2)] + \mathcal{O}(q^2)$$

$$= [\ell^2 - (\alpha+\beta)^2 m^2]$$

$\uparrow$ symmetric

In terms of l

$$N^\mu = \gamma_\alpha (\not{l} + \not{P}' + m)\, \gamma^\mu (\not{l} + \not{P} + m)\, \gamma^\alpha$$

$$P^\mu = (1-\beta) p^\mu - \alpha p'^\mu$$

$$P'^\mu = (1-\alpha) p'^\mu - \beta p^\mu$$

1) m^2 term $\quad \gamma_\alpha \gamma^\mu \gamma^\alpha = -2\gamma^\mu \quad$ drop

2) m terms

a) linear in l — drop

b) $m[\gamma_\alpha \not{P}' \gamma^\mu \gamma^\alpha + \gamma_\alpha \gamma^\mu \not{P} \gamma^\alpha] \qquad$ use $\gamma_\alpha \not{a} \not{b} \gamma^\alpha = 4\, a\cdot b$

$$= 4[P'^\mu + P^\mu] = 4m[(1-2\alpha) p'^\mu + (1-2\beta) p^\mu]$$

$$= 4m(1-\alpha-\beta)(p+p')^\mu \quad ✓$$

3) m^0 terms

a) $\gamma_\alpha \not{l} \gamma^\mu \not{l} \gamma^\alpha$ use $\int \frac{d^4 l}{(2\pi)^4} \frac{l_\mu l_\nu}{(l^2-a^2)^2} = \frac{1}{4} g_{\mu\nu} \int d^4 l \frac{l^2}{(l^2-a^2)^2}$

$\Rightarrow \frac{l^2}{4} \gamma_\alpha \gamma_\beta \gamma^\mu \gamma^\beta \gamma^\alpha = \frac{l^2}{4} \times 4 \gamma^\mu$ drop

b) linear in l - drop

c) $\gamma_\alpha \not{p}' \gamma^\mu \not{p} \gamma^\alpha = -2 \not{p} \gamma^\mu \not{p}'$ using $\gamma_\alpha \not{a} \not{b} \not{c} \gamma^\alpha = -2 \not{c} \not{b} \not{a}$

Use $(\ )\not{p}\, u(p) \to (\ ) m\, u(p)$

$-2 \not{p} \gamma^\mu \not{p}' = -2 \left[(1-\beta) \not{p} - \alpha m \right] \gamma^\mu \left[(1-\alpha) \not{p}' - \beta m \right]$

i) $\alpha \beta m^2 \gamma^\mu$ drop

ii) $\gamma^\mu \not{p}' = \{\gamma^\mu, \not{p}'\} - \not{p}' \gamma^\mu = 2 p'^\mu - m \gamma^\mu$ drop

$$\Rightarrow \not{p}\,\gamma^\mu = 2p^\mu - \gamma^\mu \cancel{\not{p}} \xrightarrow{} \text{drop}$$

m' terms $\quad 4m\left[\alpha(1-\alpha)\,p'^\mu + \beta(1-\beta)\,p^\mu\right] \longrightarrow 2m\left[\alpha(1-\alpha)+\beta(1-\beta)\right](p+p')^\mu$ ✓

iii) m^0 term $\quad -2(1-\beta)(1-\alpha)\,\underbrace{\not{p}\,\gamma^\mu \not{p}'}$

$$\underset{\uparrow}{\not{p}}\left[2p'^\mu - \not{p}'\gamma_\mu\right] = m\,2p'^\mu - (\underbrace{\{\not{p},\not{p}'\}}_{2p\cdot p'} - \underbrace{\not{p}'}_{m}\underbrace{\not{p}})\gamma_\mu \quad \text{drop}$$

$$= m\,2\,(p+p')^\mu\,(1-\beta)(1-\alpha)(-2)$$

<u>Collect:</u>

$$N^\mu = 2m(p+p')^\mu\left[2\,(1-\alpha-\beta) + \alpha(1-\alpha) + \beta(1-\beta) - 2\,(1-\beta)(1-\alpha)\right]$$

$$= 2m\,(p+p')^\mu\,(\alpha+\beta)\,(1-\alpha-\beta)$$

Do integral

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{\left(l^2 + (\alpha+\beta)^2 m^2\right)^3} = \frac{-i}{32\pi^2} \frac{1}{(\alpha+\beta)^2 m^2}$$

Such that

$$-ie\Gamma^\mu = -2e^2 \int_{Tri} d\alpha\, d\beta \; \frac{-i}{32\pi^2} \cdot 2m\,(p+p')^\mu \frac{(\alpha+\beta)(1-\alpha-\beta)}{(\alpha+\beta)^2 m^2}$$

$$= -\frac{e^2}{8\pi^2} \frac{1}{2m} (p+p')^\mu = \frac{-1}{2m} (p+p')^\mu \times \frac{\alpha}{2\pi}$$

$$\Longrightarrow F_2(0) = \frac{\alpha}{2\pi} = (g-2)$$

✓

$$\int d\alpha\, d\beta \frac{1}{(\alpha+\beta)} = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{1}{(\alpha+\beta)} = \int_0^1 d\alpha \ln \tfrac{1}{\alpha} = 1$$