Note Title 12/1/2009

Supersymmetry overview #1

Weyl spinors – 2 component

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$$

Dirac $\psi = \begin{pmatrix} u \\ \ell \end{pmatrix} \Longrightarrow P_L \psi = \frac{1}{2}\begin{pmatrix} u - \ell \\ -(u-\ell) \end{pmatrix} = \begin{pmatrix} \xi \\ -\xi \end{pmatrix}$ $\qquad P_R \psi = \frac{1}{2}\begin{bmatrix} (u+\ell) \\ (u+\ell) \end{bmatrix} = \begin{bmatrix} \chi \\ \chi \end{bmatrix}$

ξ, χ = 2 component

define $\sigma^\mu = (1, -\vec{\sigma})$

$\bar{\sigma}^\mu = (1, \vec{\sigma})$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$$

$$= \underline{\xi^\dagger i \sigma^\mu \partial_\mu \xi} + \underline{\chi^\dagger i \bar{\sigma}^\mu \partial_\mu \chi} - m\left[\xi^\dagger \chi + \xi^\dagger \chi^\dagger\right]$$

Wess Zumino model

– complex scalar + Weyl spinor ψ

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) + \psi^\dagger i \sigma^\mu \partial_\mu \psi$$

Supersymmetry

$$\delta\phi = \varepsilon \psi \qquad \delta \phi^* = \varepsilon^\dagger \psi^\dagger$$

$$\delta \mathcal{L}_\phi = \varepsilon \partial_\mu \phi^* \partial^\mu \psi + \varepsilon^\dagger \partial_\mu \psi^\dagger \partial^\mu \phi$$

$\delta\psi$ involves $\partial_\mu \phi$

$$\delta \psi^\dagger = i \sigma^\mu \varepsilon \partial_\mu \phi^* \qquad \delta\psi = -i \sigma^\mu \varepsilon^\dagger \partial_\mu \phi$$

$$\Longrightarrow \delta \mathcal{L}_{TOT} = 0$$

$$\left.\begin{aligned} \delta_\varepsilon \phi &= \varepsilon \psi \\ \delta_\varepsilon \psi &= -i \sigma^\mu \varepsilon \partial_\mu \phi \end{aligned}\right\} \quad \text{SUSY invariance}$$

Do not commute

$$(\delta_{\varepsilon_2} \delta_{\varepsilon_1} - \delta_{\varepsilon_1} \delta_{\varepsilon_2}) \phi = (\varepsilon_1 \sigma^\mu \varepsilon_2^+ - \varepsilon_2 \sigma^\mu \varepsilon_1^+) \underbrace{i \partial_\mu}_{*} \phi$$

$$(\qquad\qquad) \psi = (\qquad\qquad) \overset{*}{i \partial_\mu} \psi \quad \text{if use Dirac eq.}$$

SUSY Currents & Charges

$$J^\mu_\alpha = (\sigma^\nu \bar\sigma^\mu \psi)_\alpha \partial_\nu \phi^*$$ 2 component $\alpha = 1, 2$

$$J^{\dagger\mu} = (\bar\psi \bar\sigma^\mu \sigma^\nu) \partial_\nu \phi$$

$$Q_\alpha = \int J^0_\alpha \, d^3x \, \sqrt{2}$$

T^{0i}

Also define $P^\mu = (H, \vec{P})$

$$\int d^3x \left[\Pi^*_\phi \Pi_\phi + \nabla\phi^* \nabla\phi \; --- \right.$$

SUSY algebra

$$\{Q, Q^\dagger\} = 2\sigma^\mu P_\mu$$

$$\{Q, Q\} = 0$$

$$\{Q^\dagger, Q^\dagger\} = 0$$

$$[Q, P_\mu] = 0 \quad \leftarrow \quad Q, E, \vec{p} \text{ all conserved}$$

- SUSY algebra mixes Spacetime symmetries
- generalize algebra to interacting theories

Auxillary Fields

add $\mathcal{L} = F^* F$ $\qquad$ E.L $\Rightarrow F = 0$

Then $\delta F = \xi^\dagger \bar{\sigma}^\mu i \partial_\mu \psi$

We get $(\delta_{\xi_1} \delta_{\xi} - \delta_{\xi_2} \delta_{\xi_1}) \psi = (\qquad) i \partial_\mu \psi$ without using Dirac eq

D.O.F.

$\phi = 2$ D of F , $\psi = 4$ D o F

but $i \bar{\sigma}^\mu \partial_\mu \psi = 0 \xRightarrow{\text{reduces}} 2$ D.O.F.

without Dirac eq

$\phi = 2$, $F = 2$, $\psi = 4$ $\Rightarrow$ D. of F match

Eq of motion remove F , $\psi \rightarrow 2$ D of F.

Superspace $(x, \theta, \bar{\theta})$

$\Rightarrow$ 4 new coord θ_α, $\bar{\theta}$

$\alpha = 1, 2$

Grassmann $\{\theta, \bar{\theta}\} = 0 =$

Superfields $\Phi(x, \theta, \bar{\theta})$

Chiral Superfield $\Phi = \Phi(x, \theta)$ no $\bar{\theta}$

$$\theta^2 = \theta^a \theta^b \varepsilon_{ab} \qquad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

$$\Phi = \phi(x) + \sqrt{2}\,\theta^\alpha \psi_\alpha + \theta^2 F$$

$$\delta_\varepsilon \Phi = \left[\frac{\partial}{\partial \theta} + 2i\theta\, \sigma^\mu \partial_\mu \right] \Phi$$

$\Phi^+ \leftarrow$ right chiral (technology)

End result

$$S = \int d^4x \, d^2\theta \, d^2\bar{\theta} \, \Phi^+ \Phi = \int d^4x \, \mathcal{L}_{WZ}$$

Vector Superfield $\quad A^a_\mu \,,\; \lambda^a$ ← Spin $\frac{1}{2}$ $\qquad$ aux fields

$$\mathcal{L}_G = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \lambda^{a\dagger} i \bar{\sigma}^\mu D^{ab}_\mu \lambda^b + \frac{1}{2} D^a D^a$$

← in adjoint $\quad D^{ab}_\mu = \partial_\mu \delta^{ab} + i g f^{abc} A^c_\mu$

SUSY trans

$$\delta A^a_\mu = \frac{1}{\sqrt{2}} \left(\varepsilon^\dagger \bar{\sigma}_\mu \lambda^a + \lambda^a \sigma_\mu \varepsilon \right)$$

$$\delta \lambda^a = \qquad F_{\mu\nu} \qquad + \qquad \varepsilon D^a$$

$$\delta D^a = \cdot \varepsilon \bar{\sigma}^\mu D_\mu \lambda^a$$

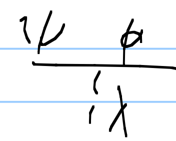
Superfield notation

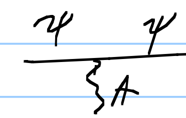
$$t^a D^a$$
$$\Downarrow$$

$$V(x,\theta,\bar\theta) = \theta\sigma_\mu\bar\theta A^\mu + i\,\theta\theta\,\bar\theta\bar\lambda - \bar\theta\bar\theta\,\theta\lambda + \frac{1}{2}\theta\theta\bar\theta\bar\theta D$$

Generates gauge interactions

$$\int d^2\theta\, d^2\bar\theta\, \Phi^\dagger e^{2gV}\Phi = \mathcal{L}_{WZ}(\partial_\mu \to D_\mu) + g\sqrt{2}(\phi^*\lambda\psi - \bar\lambda\bar\psi\phi)$$





$$+\, g\,\underbrace{\phi^* D\phi}_{*}$$

Superpotential

– need Yukawa $\phi\bar{\psi}\psi$

Look at $\quad\swarrow \Phi(x,\theta)$

$$\int d\theta^2\, \Phi_1 \Phi_2 \Phi_3 \sim \int d^2\theta \Big[\underbrace{\phi_1\, \theta\psi_2\, \theta\psi_3}_{\text{Yukawa}} + \underbrace{\phi_1 \phi_2 F_3\, \theta\theta}_{F\phi\phi} \cdots$$

Superpotential – only θ

$$W[\Phi_i] = L^i \Phi_i + \frac{1}{2} m^{ij} \Phi_i \Phi_j + g^{ijk} \Phi_i \Phi_j \Phi_k$$

$$\mathcal{L} = \int d^2\theta\, W[\Phi] = \frac{\partial W}{\partial \phi_i} F^i - \frac{1}{2} \frac{\partial W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j$$

$\uparrow$ F term $\qquad \uparrow$ masses + Yukawas

D terms + F terms

- after interactions

$$\mathcal{L} = -F_i^\dagger F^i + \frac{\partial W}{\partial \phi_i} F^i + \frac{1}{2} D^a D^a + g\phi^\dagger D \phi$$

Integrate out D + F

$$F_i^\dagger = \frac{\partial W}{\partial \phi_i} \quad , \quad D^a = g\,\phi^* t^a \phi$$

Substitute back in

$$\mathcal{L} = \underbrace{\left(\frac{\partial W}{\partial \phi_i}\right)^\dagger \left(\frac{\partial W}{\partial \phi_i}\right)}_{F \text{ terms}} \mp \frac{1}{2} g^2 \underbrace{\phi^* t^a \phi\, \phi^* t^a \phi}_{\lambda \phi^4}$$

← real potential for remaining fields

$m^2 \phi^* \phi$

$g^2 (\phi^* \phi)^2 \cdots + g \bar{\psi} \psi \phi$

Model Building

1) Choose # + type of chiral superfields (ϕ_i, ψ_i, F_i)
vector (λ^a, A^a_μ, D)

2) Most general superpotential

$$W = L\Phi + m^{ij}\Phi^i\Phi^j + g^{ijk}\Phi_i\Phi_j\Phi_k$$

3) Write KE

$$\int d^2\theta\, d^2\bar{\theta}\ \Phi^\dagger e^{i2gV}\Phi = KE + \text{gauge} + \text{gaugino} + D\ \text{Term}$$

4) Potential from Superpotential

$$\int d^2\theta\ W = \frac{\partial W}{\partial\phi_i}F^i + \psi_i\psi_j\frac{\partial^2 W}{\partial\phi_i\phi_j}$$

5) Integrate D & F

$$\mathcal{L} = \frac{\partial W}{\partial \phi^i} \frac{\partial W}{\partial \phi^i} + \frac{1}{2} g \left(\phi^+ t^a \phi \right)^2$$