Standard Model 4

Note Title 11/19/2009

Pion/Kaons in QCD

if $m_q \to 0$ $\quad \psi_L \to L\,\psi_L$

$\psi_R \to R\,\psi_R$

don't see partners

– Symmetry hidden $\Rightarrow$ Goldstone bosons

$P \quad \& \quad P + \pi(q=0)$

Dynamical Symmetry Breaking

$$\Rightarrow \quad \mathcal{L} = \frac{F^2}{4}\,\mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^+\right) + \ldots$$

interesting

External source technique

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} - m)\psi$$

SU(3) in color

$$\not{D} = \left(\not{\partial} + ig\frac{\lambda^a}{2}\not{G}^a\right)\mathbb{1}$$

in flavor

$$\bar{\psi} m \psi = \bar{\psi}_L m \psi_R + \bar{\psi}_R m \psi_L \Rightarrow m = \begin{pmatrix} m_u & & 0 \\ & m_d & \\ 0 & & m_s \end{pmatrix}$$

Add external sources

$l_\mu,\ r_\mu,\ s,\ p$ ← 3×3 in flavor space

$$s = s^0 + t^A s^A$$

SU(3) matrix in flavor SU(3)

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}_L\, i\not{D}_L\, \Psi_L + \bar{\Psi}_R\, i\not{D}_R\, \Psi_R + \bar{\Psi}_L (s + ip)\Psi_R + \bar{\Psi}_R (s - ip)\Psi_L$$

$$\not{D}_L = \left(\partial_\mu + ig\frac{\lambda^a G^a}{2}\right)\mathbb{1} + i\,l_\mu(x)$$

$$\not{D}_R = (\qquad\qquad)\mathbb{1} + i\,r_\mu(x)$$

Pure QCD $\Rightarrow$ $l_\mu = r_\mu = 0$, $S = M$, $p = 0$

QED + photon $\quad l_\mu = r_\mu = eQA_\mu$, $S = m$, $p = 0$

$$\tau\begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$$

QCD + W $\quad r_\mu = 0$, $l_\mu = \frac{g_2}{\sqrt{2}}\left(W_\mu^+\begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + h.c.\right)$

Currents

$$\frac{\partial \mathcal{L}}{\partial S^a} = \bar{\psi}\psi$$

$$\frac{\partial \mathcal{L}}{\partial l_\mu^a} = \bar{\psi}_L \gamma_\mu t^a \psi_L$$

$$\frac{\partial \mathcal{L}}{\partial A^\mu} = \bar{\psi}\gamma^\mu Q \psi = J_\mu^{em}$$

Enhanced Symmetry

$$\psi_L \to L\psi_L \qquad\qquad + \qquad (s+ip) \longrightarrow L(s+ip)R^\dagger$$
$$\psi_R \to R\psi_R \qquad\qquad \bar\psi_L(s+ip)\psi_R \text{ - invariant}$$

exactly invariant

make it $SU(3)_L \times SU(3)_R$ gauge symmetry $\quad l_\mu \to L(l_\mu + iL^\dagger\partial_\mu L)L^\dagger$

$$\bar\psi_L \; \not{D}_L \psi_L \Rightarrow \bar\psi L^\dagger\left(\not\partial + iL(\not{l}_\mu + iL^\dagger \not\partial_\mu L)L^\dagger\right) L\psi_L$$
$$= \bar\psi\left[\cancel{(L^\dagger \not\partial L)} + i\not\partial + i\not{l} \cancel{- i L^\dagger \not\partial L}\right]\psi_L = \bar\psi_L \not{D}_L \psi_L$$

$$D_{\mu L} \to L D_\mu L^\dagger$$

$$\psi_L \to L\,\psi_L \quad , \quad l_\mu \to L[\qquad]L^\dagger$$
$$\psi_R \to R\,\psi_R \qquad\quad r_\mu \to R[\qquad]R^\dagger$$
$$(s+ip) \to L\,(s+ip)\,R^\dagger$$

$$[D_{\mu L}, D_{\nu L}] = \partial_\mu l_\nu - \partial_\nu l_\mu + [l_\mu, l_\nu] = L_{\mu\nu}$$
$$L_{\mu\nu} \to L\,L_{\mu\nu}\,L^\dagger$$
$$R_{\mu\nu} \to R\,R_{\mu\nu}\,R^\dagger$$

Go to path integral

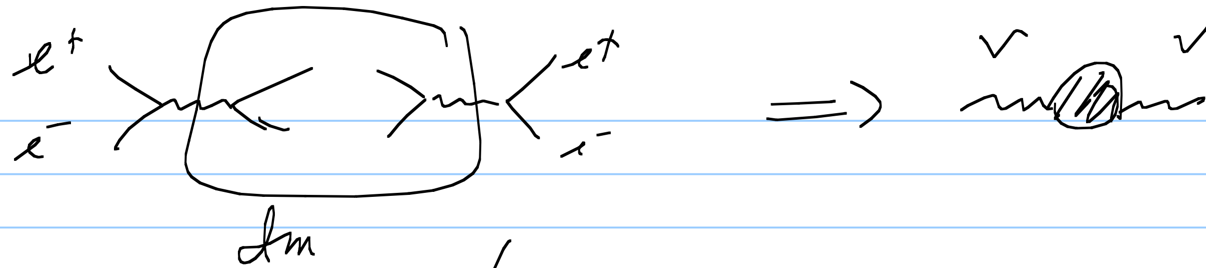
$$Z[s,p,l,r] = \int dA_\mu\, d\psi\, d\bar\psi\; e^{i\int d^4x\, \mathcal{L}(\psi, A, s, p, l, r)}$$

$$\langle 0 | \bar\psi \psi | 0 \rangle = \frac{i}{Z}\frac{\delta Z}{\delta s}$$

* signal of chiral symmetry breaking (equiv of v in SSB)

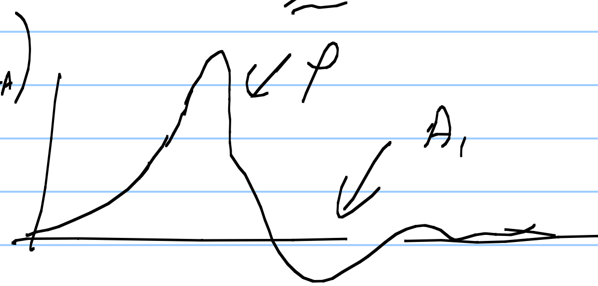
$$\langle 0 | \bar\psi \frac{t^a}{2} \gamma_\mu (1+\gamma_5) \psi(x) \;\; \bar\psi \frac{t^b}{2} \gamma_\nu (1+\gamma_5) \psi(y) | 0 \rangle = \frac{(-i)^2}{Z} \frac{\delta Z}{\delta l^a_\mu(x)\, \delta l^b_\nu(y)}$$

$$\frac{\delta}{\delta l\, \delta r} = \langle \bar\psi \gamma^\mu (1+\gamma_5) \psi \;\; \bar\psi \gamma_\nu (1-\gamma_5) \psi(y) \rangle = 0 \text{ if no } \chi SB$$



$\tau \to \cancel{\nu}$ Hadron

$\mathrm{Im}(\Pi_{VV} - \Pi_{AA})$



Effective $\mathcal{L}$

- only d.o.f π, K

Match

$$Z[s+p, l, r] = \int dA\, d\psi\, d\bar{\psi}\; e^{i\int d^4x\, \mathcal{L}_{QCD}}$$

$$= \int d\pi\; e^{i\int d^4x \left[\mathcal{L}_{eff}(\pi, \sigma, p, l, r)\right]}$$

exact SU(3) gauge sym

Global SU(3)

$$U = \exp i \frac{t^a \phi^a}{F}$$

$$U \to L U R^\dagger$$

$$\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \; -$$

Gauge symm.

$$D_\mu U \to L (D_\mu U) R^\dagger$$

Solution

$$D_\mu U = \partial_\mu U + i l_\mu U - i U r_\mu$$

$L_{\mu\nu}$, $R^{\mu\nu}$

$s + ip \Longrightarrow \chi = 2B_0 (s + ip) \Longrightarrow \chi \to L \chi R^\dagger$

$\partial_\mu \sim E$

$l_\mu \sim E$

$L_{\mu\nu} \sim E^2$

and $\chi \sim E^2$

after $s \to m$

$m_\pi^2 = 2B_0 m$

Constructing $\mathcal{L}_{eff}$

E^0 , $U^\dagger U = 1$

E^2 , $\mathrm{Tr}\left(D_\mu U (D_\mu U)^\dagger\right)$, $\mathrm{Tr}(\chi^\dagger U + U^\dagger \chi)$

$$\mathcal{L}_2 = \frac{F^2}{4} \mathrm{Tr}(D_\mu U D^\mu U^\dagger) + \frac{F^2}{4} \mathrm{Tr}(\chi^\dagger U + U^\dagger \chi) \qquad (F, B_0)$$

$2B_0 s$

$O(E^4)$

$$\mathcal{L} = L_1 \left[\mathrm{Tr}\, D_\mu U D^\mu U^\dagger\right]^2 \cdots\cdots \mathrm{Tr}(\chi U + U^\dagger \chi)\, \mathrm{Tr}(D_\mu U D^\mu U^\dagger)$$

$$+ L_{10} \mathrm{Tr}(L_{\mu\nu} U R^{\mu\nu} U^\dagger)$$

$+10$ terms

Explore I

Tree level (no loops)

1) $\langle 0|\bar{\psi}\psi|0\rangle = \dfrac{\delta Z}{\delta s^0} = -F^2 B_0 \qquad B_0 = -\dfrac{1}{F}\langle 0|\bar{\psi}\psi|0\rangle$

2) π & K masses

$$m_\pi^2 = B_0(m_u + m_d) = 2B_0\bar{m}$$

$$m_{K^+}^2 = B_0(m_u + m_s)$$

$$m_{K^0}^2 = B_0(m_d + m_s)$$

$$m_{\eta_8}^2 = B_0\frac{1}{3}(m_u + m_d + 4m_s) \simeq \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2$$

$$\frac{\hat{m}}{m_s} = \frac{m_\pi^2}{2m_K^2 - m_\pi^2} = \frac{1}{26}$$

$$\frac{m_d - m_u}{m_d + m_u} = \frac{m_{K^0}^2 - m_{K^+}^2}{m_\pi^2} = 0.29 \Rightarrow \frac{m_u}{m_d} = 0.55$$

remove EM

Complete $U(1)_A$ story

IF we had $U(3)_A$ DSB $\Rightarrow$ 9 G.B.

Then masses

$$m^2 = B_0 \begin{pmatrix} 2\hat{m} & 0 & & 0 \\ 0 & m_s+\hat{m} & & \\ & & \frac{2}{3}(2m_s+\hat{m}) & \frac{2\sqrt{2}}{3}(\hat{m}-m_s) \\ & 0 & \frac{2\sqrt{2}}{3}(\hat{m}-m_s) & \frac{2}{3}(m_s+2\hat{m}) \end{pmatrix}$$

$$\Rightarrow m_a^2 = \underline{\underline{m_\pi^2}} \quad , \quad m_b^2 = 2m_K^2 - m_\pi^2$$

$U(1)$ problem state degenerate with pions

After $U(1)_A$ anomaly

η_0  η_0

$$m = \begin{pmatrix} \text{same} & m_s - \hat{m} \\ m_s - \hat{m} & + \varepsilon \end{pmatrix}$$

$\varepsilon \sim O(1\,\text{GeV})$

π, K, η_8, η_0

$\eta_8 \leftarrow \eta(549)$, $\eta_0 \leftarrow \eta'(960)$

$\eta(549) \leftrightarrow \eta'(960)$ mixing