Standard Model 2

Note Title 11/12/2009

$$\mathcal{L} = -\frac{1}{4}\underset{U(1)}{F^2} - \frac{1}{4}F^2 - \frac{1}{4}F^2 + \bar{\Psi}\, i \not{D}\, \Psi$$

only on left

$$D_\mu \Psi = \left[\partial_\mu + i g_1 \frac{Y}{2} B_\mu + i g_2 \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu (1+\gamma_5) + i g_3 \frac{\lambda}{2}\cdot G_\mu\right]\Psi$$

odd set

ϕ = Higgs – $SU(2)$ doublet, Hypercharge $= +1$, no $SU(3)$ coupling

$$D_\mu \phi = \left[\partial_\mu + i g_1 \frac{1}{2} B_\mu + i g_2 \frac{\vec{\tau}}{2}\cdot\vec{W}\right]\phi$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$ electric charge

$\Longrightarrow \phi^+, \phi^-, \phi^0, \bar{\phi}^0$

$$V(\phi) = -\mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

if $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_1 + i\pi_2 \\ \sigma + i\pi^3 \end{pmatrix} \Rightarrow \phi^*\phi = \frac{1}{2}(\sigma^2 + \vec{\pi}^2)$

$= \sigma$ model

<u>Story</u>

minima at $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

Will end up $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ (v + H) \end{pmatrix} e^{i \vec{\tau} \cdot \vec{\pi} / v}$

Ground state Masses of gauge bosons

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi) \qquad \phi(1/\sqrt{2})^2$$

$$= (0, v)\left(\partial_\mu - i\frac{g_1}{2}B_\mu - i g_2 \frac{\vec{\tau}}{2}\cdot\vec{W}\right)\left(\cancel{\partial_\mu}^{\,0} + i\frac{g_1}{2}B_\mu + i g_2\frac{\vec{\tau}}{2}\cdot\vec{W}\right)\begin{pmatrix}0\\ v\end{pmatrix} \quad \frac{1}{2}$$

$$= \left(\frac{g_2 v}{2}\right) W^+_\mu W^{-\mu} + \frac{v^2}{8}\left(W^3_\mu, B_\mu\right)\begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2\end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu\end{pmatrix}$$

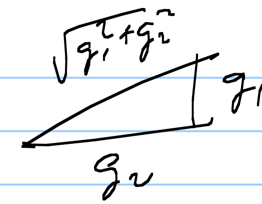
$$\det\begin{pmatrix} g_2^2-\lambda & -g_1 g_2 \\ -g_1 g_2 & g_1^2-\lambda\end{pmatrix} = -\lambda(g_1^2+g_2^2) + \lambda^2 \Rightarrow \begin{array}{l}\lambda = 0 \\ \lambda = g_1^2+g_2^2\end{array}$$

$$\Rightarrow M_{W^\pm} = \frac{g_2 v}{2}, \quad M_Z = \frac{v}{2}(g_1^2+g_2^2)^{1/2}, \quad M_\gamma = 0$$

define $\cos\theta_W = \dfrac{g_2}{(g_1^2+g_2^2)^{1/2}} \Rightarrow M_W = M_Z \cos\theta_W$

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$

$$A_\mu = \sin\theta_W B_\mu + \cos\theta_W B_\mu$$



$$\tan\theta_W = \frac{g_1}{g_2}$$

From weak decay

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{1}{2v^2} \Rightarrow v = 246 \text{ GeV}$$

$$v = \sqrt{\frac{\mu^2}{\lambda}} \quad , \quad M_H = 2\mu^2 = 2v^2 \times \lambda$$

$$V(H) = 2\mu^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

Rest of story

4 Gauge bosons $\Rightarrow$ 8 DofF
4 Scalars $\Rightarrow$ 4 "

end with 3 massive + 1 massless $\Rightarrow$ 11 DofF
H $\Rightarrow$ 1 Dof

$$\phi = e^{i\vec{\tau}\cdot\vec{\pi}/v} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

Then

$$D_\mu \phi = e^{i\vec{\tau}\cdot\vec{\pi}/v} \left[\partial_\mu + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu + \vec{\tau} \cdot \frac{\partial_\mu \vec{\pi}}{v} \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

do gauge change $W_\mu^a \to W_\mu^a - \frac{\partial_\mu \pi^a}{g v}$

$\Rightarrow$ field disappears completely

Interactions - fermions

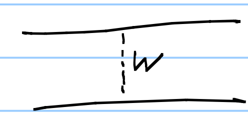
$$\mathcal{L} = \bar{\Psi}_i \left(i\not\partial - g_1 \frac{Y_i}{2} \not{B}^{\mu} - \frac{g_2}{2} \vec{\tau} \cdot \vec{\not{W}} \right) \Psi_i$$

Charged int

$$\mathcal{L} = \frac{g}{\sqrt{8}} \left(W^+_\mu J^{ch\,\mu} + W^-_\mu J^{+ch\,\mu} \right)$$

$$J^{ch}_\mu = \left(\bar{\nu} \gamma^\mu (1+\gamma_5) e + \bar{u} \gamma^\mu (1+\gamma_5) d \qquad \right)$$

$$\mathcal{H}_W = \underbrace{-\frac{g_2^2}{8 M_W^2}}_{\frac{G_F}{\sqrt{2}}} J^{ch}_\mu J^{+ch\,\mu}$$



Neutral sector

$$\mathcal{L} = -g_3 W^3_\mu \bar{J}^{(3)}_{L\mu} - g_1 B_\mu \bar{J}^{(1)}_\mu = \underline{g_1 \cos\theta_W} A^\mu J^\mu_{em} - \frac{g_2}{2\cos\theta_W} J^Z_\mu Z^\mu$$

$$J^\mu_{em} = -\left[\bar{e}\gamma_\mu e + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d\right]$$

$$J^Z_\mu = \bar{u}\left[\frac{1}{2}\gamma_\mu(1+\gamma_5) - \frac{4}{3}\sin^2\theta_W\gamma_\mu\right]u + \bar{d}\left[-\frac{1}{2}\gamma_\mu(1+\gamma_5) + \frac{2}{3}\sin^2\theta_W\right]d$$

$$+ \underbrace{\bar{\nu}\frac{1}{2}\gamma_\mu(1+\gamma_5)\nu}_{\text{only LH}} \qquad + \bar{e}\underbrace{\left[-\frac{1}{2}\gamma_\mu(1+\gamma_5) + 2\sin^2\theta_W\right]}_{\text{Vector } -\frac{1}{2}(1-4\sin^2\theta_W)}e$$

$\uparrow \simeq 1/4$

$$= \bar{\Psi}\left[T_3\gamma_\mu(1+\gamma_5) - 2Q\sin^2\theta_W\right]\Psi$$

$Q = T_3 + \frac{Y}{2}$ from before

$\underline{Q = \frac{Y}{2} \text{ for RH field}}$

$e = g_1 \cos\theta_W$

<u>For electron</u>

$B_\mu = \cos\theta A_\mu - \sin\theta Z_\mu$, $W_3 = \sin\theta A_\mu + \cos\theta Z_\mu$

$$\mathcal{L} = \bar{e}_L\left[-\frac{g_1}{2} B_\mu - \frac{g_2}{2} W_\mu^3\right] e_L + \bar{e}_R\left[-g_1 B_\mu\right] e_R$$

$$= A_\mu\left[-g_1 \cos\theta_W \bar{e}_R e_R - \underbrace{\left(\frac{g_1}{2} \cos\theta_W + \frac{g_2}{2} \sin\theta_W\right)}_{\text{same}} \bar{e}_L e_L\right]$$

$$\tan\theta_W = \frac{g_1}{g_2} \Rightarrow g_2 \sin\theta_W = g_1 \cos\theta_W$$

$$= -g_1 \cos\theta_W A_\mu\left[\bar{e}_R e_R + \bar{e}_L e_L\right]$$

Prelims:

ϕ is $SU(2)$ doublet

$\bar{\phi} = i\tau_2 \phi^*$ is a $SU(2)$ doublet

Normally $\phi \sim N$ in $SU(N)$, $\phi^* \sim \bar{N}$ of $SU(N)$

$\Rightarrow \bar{2} = 2$ in $SU(2)$

To see this

$$\phi \to \phi' = U\phi \qquad U = e^{i\vec{\alpha}\cdot\vec{\tau}}$$

$$\bar{\phi} \to \bar{\phi}' = i\tau_2 (U^*\phi^*) = \tau_2 U^* \tau_2 (i\tau_2\phi^*) = \underbrace{\tau_2 U^* \tau_2}_{\stackrel{?}{=} U} \bar{\phi}$$

use

$$\left.\begin{aligned} \tau_2 \tau_1^* \tau_2 &= \tau_2 \tau_1 \tau_2 = -\tau_1 \\ \tau_2 \tau_2^* \tau_2 &= -\tau_2 \tau_2 \tau_2 = -\tau_2 \\ \tau_2 \tau_3^* \tau_2 &= -\tau_3 \end{aligned}\right\} \quad \tau_2 \tau_i^* \tau_2 = -\vec{\tau}$$

$$\tau_2 \left(e^{i\vec{\alpha}\cdot\vec{\tau}}\right)^* \tau_2 = \tau_2\, e^{-i\vec{\alpha}\cdot\vec{\tau}^*}\, \tau_2 = e^{i\alpha\cdot\vec{\tau}} = U \quad \checkmark$$

In SU(3)

$$\lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\lambda^2 = \begin{pmatrix} \tau^2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\lambda^2 \lambda^{8*} \lambda^2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & 0 \\ 0 & 1 & \\ & & 0 \end{pmatrix} \neq \lambda^8$$

$$\bar{\phi} = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$$

Yukawa couplings

$(\overline{\Psi}_{q_L}\ \phi)\ d_R$; $(\overline{\Psi}_{\ell L}\ \phi)\ e_R$

Y $\quad -\frac{1}{3} \quad 1 \quad -\frac{2}{3}$ $\qquad\qquad 1 \quad 1 \quad -2$

$d_R \to e^{-i\frac{\alpha}{3}}\, d_R$, $\Psi_q \to e^{+\frac{2}{3}i\alpha}\, \Psi_q$ $\qquad \phi \to e^{i\alpha}\, \phi$

$\Psi \to e^{iY\alpha}\, \Psi$

Also:

$(\overline{\Psi}_{q_L}\ \overline{\phi})\ u_R$ $\qquad (\overline{\Psi}_{\ell L}\ \overline{\phi})\ \nu_R$

Y $\quad -\frac{1}{3} \quad -1 \quad \frac{4}{3}$ $\qquad\qquad 1 \quad -1 \quad 0$

$$\mathcal{L}_Y = -\Gamma^{(d)}_{ij}\,\overline{\psi}_{qL_i}\,\phi\,d_{jR} - \Gamma^{(u)}_{ij}\,\overline{\psi}_{qL_i}\,\overline{\phi}\,u_{R_j}$$

$$-\Gamma^{e}_{ij}\,\overline{\psi}_{\ell L_i}\,\phi\,e_{R_j} - \Gamma^{(\nu)}_{ij}\,\overline{\psi}_{\ell L_i}\,\overline{\phi}\,\nu_{R_j} + h.c.$$

When $\phi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$ $\qquad \overline{\psi}_{qL}\,\phi\,d_R = \frac{1}{\sqrt{2}}(v+H)\,\overline{d}_L d_R$

$\overline{\phi} \to \frac{1}{\sqrt{2}}\begin{pmatrix} v+H \\ 0 \end{pmatrix}$ $\qquad \overline{\psi}_{qL}\,\overline{\phi}\,u_R = \frac{1}{\sqrt{2}}(v+H)\,\overline{u}_L u_R$

Mass matrix

$$\mathcal{L} = \frac{-1}{\sqrt{2}}(v+H)\Big[\,\overline{d}_L\Gamma^{d}d_R + \overline{d}_R\Gamma^{+d}d_L + \overline{u}_L\Gamma^{u}u_R + h.c.$$

$$+\,\overline{e}_L\Gamma^{e}e_R + h.c. + \overline{\nu}_L\underbrace{\Gamma^{\nu}}_{3\times 3}\nu_R + h.c.\,\Big]$$