Renormalization 5

Note Title 4/6/2010

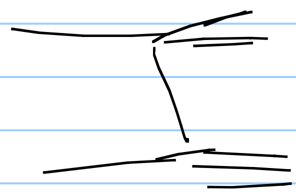
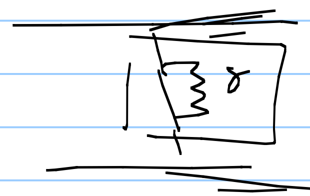
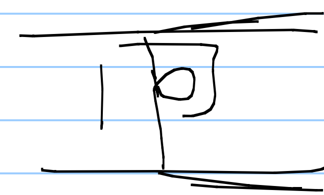
Renorm

1) Coupling constant $\lambda, e, \ldots$
2) Mass renorm. $\Big\}$ Self energy diagrams
3) Wavefunction renorm.

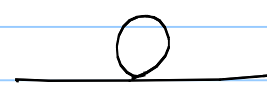
Propagator

$$\frac{i}{p^2 - m^2 + i\varepsilon} = \frac{i}{E^2 - \vec{p}^{\,2} - m^2 + i\varepsilon} = i\Big[\underbrace{\frac{P}{E^2 - \vec{p}^{\,2} - m^2}}_{\text{off shell}} - i\pi\,\underbrace{\delta(E^2 - \vec{p}^{\,2} - m^2)}_{\text{on shell}}\Big]$$

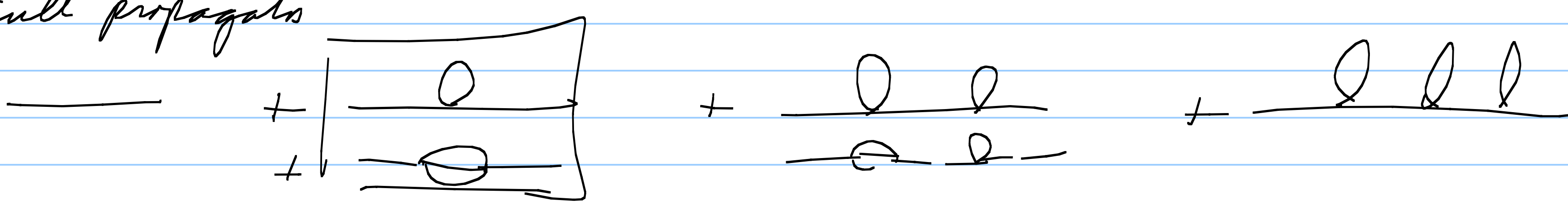
Interactions


+

or


Self energies

$-i\,\Sigma(p) \equiv$  or  (drop external factors $\bar{u}(\ )u$)

Full propagator



$$\frac{i}{p^2 - M_0^2 + i\varepsilon} + \frac{i}{p^2 - M_0^2 + i\varepsilon}\,(-i\Sigma(p))\,\frac{i}{p^2 - M_0^2 + i\varepsilon} + \frac{i}{p^2 - M_0^2 + i\varepsilon}\,(-i\Sigma)\,\frac{i}{p^2 - M_0^2 + i\varepsilon}\,(-i\Sigma)\,\frac{i}{p^2 - M_0^2 + i\varepsilon} + \cdots$$

$$= \frac{i}{p^2 - M_0^2 - \Sigma(p) + i\varepsilon}$$

Now pole occurs at

$$p^2 - M_0^2 - \Sigma(p) = 0$$

In Pert theory

$$\Sigma(p^2) = \Sigma(m^2) + (p^2 - m^2)\,\Sigma'(m^2) + \cdots$$

$$m^2 = M_0^2 + \Sigma(m^2) = \text{physical mass} \quad \Leftarrow \text{pole}$$

Wavefunction renorm

$$\frac{i}{p^2 - m_0^2 - \Sigma(m^2) - \underline{(p^2-m^2)\,\Sigma'(m^2)}} = \frac{i}{(p^2-m^2)(1-\Sigma'(m))} = \frac{iZ}{p^2-m^2+i\epsilon}$$

Logic

$$i\,D_F(x-y) = \langle 0 | T\,\phi(x)\,\phi(y) | 0\rangle$$

$$(\Box + m^2)_y\, i\,D_F(x-y) = \langle 0 | \underbrace{[\phi(x), \partial_0 \phi(y)]}_{\text{defining property}}{}_{x_0=y_0} | 0\rangle\, \delta(x_0-y_0) = i\,\delta^4(x-y)$$

↑ no factor of Z

Need to readjust norm $\phi_r = Z^{1/2}\phi$

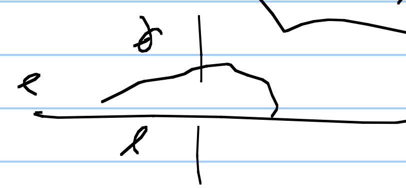
ϕ_r will have correct propagator

In QM

$$|\psi_i\rangle = N\left[|\psi_{i0}\rangle + \sum_{j\neq i} |\psi_{j0}\rangle \frac{\langle\psi_{j0}|V|\psi_{i0}\rangle}{E_i - E_j}\right]$$

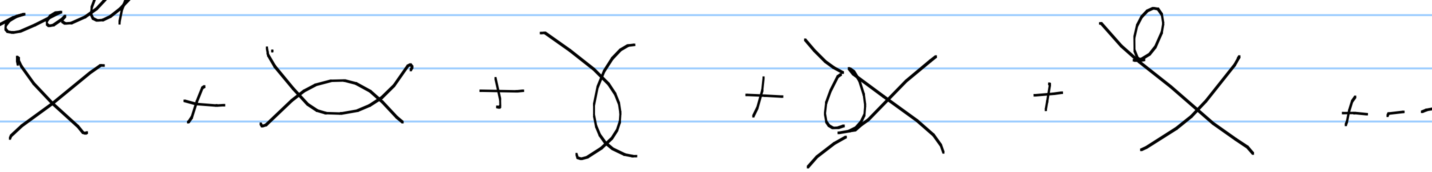
$Z^{1/2}$

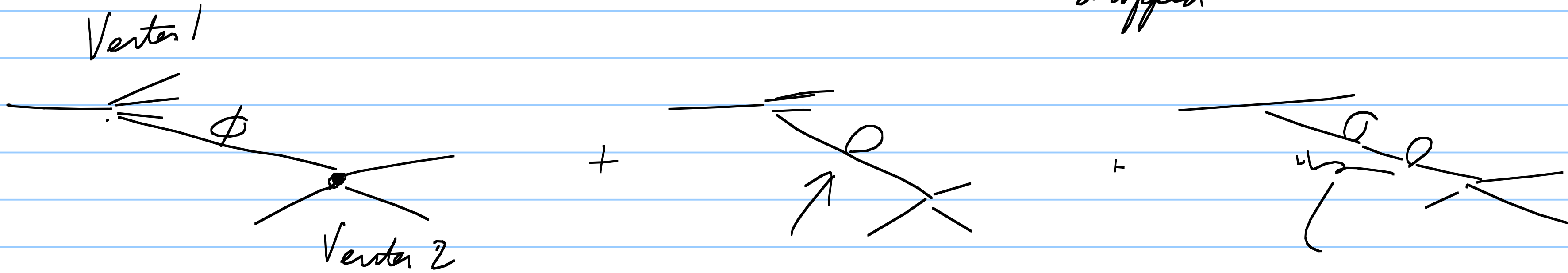
$$\langle\psi_i|\psi_i\rangle = \underline{\qquad\qquad}$$



Logic for dropping Self energies

Recall





net effect is just going to physical mass

On shell part $\quad$ (Vertex 1) $\rightarrow \pi\,\delta(p^2 - M_{phys}^2)\,Z$ (Vertex #2)

some effect of Z

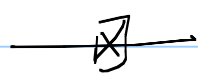
Formal technique - mass renorm by itself

- counterterm method

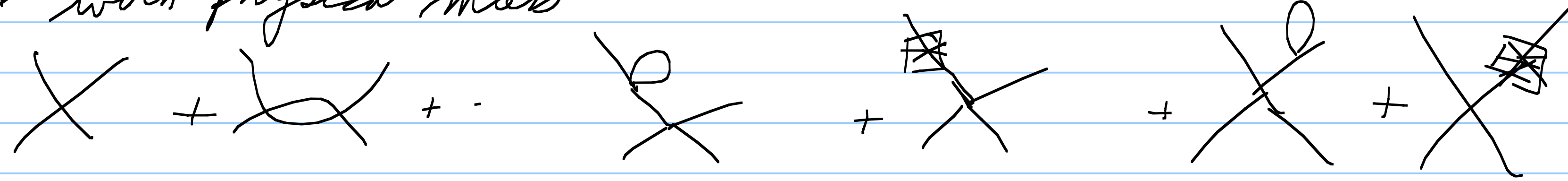
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4$$

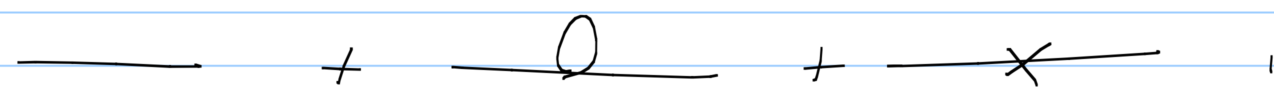
Express in terms of physical mass

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 + \underbrace{\frac{\overbrace{\delta m^2}^{m^2 - m_0^2}}{2}\phi^2}_{\text{Treat as perturbation}}$$

- Feynman rule  $= i\,\delta m^2$

Calculate with physical mass

$-i\mathcal{M} =$ 

$-i\,\Sigma(p^2) =$ 

Propagator

$$= \frac{i}{p^2-m^2+i\varepsilon} + \frac{i}{p^2-m^2}\,(-i\Sigma)\,\frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\,i\delta m^2\,\frac{i}{p^2-m^2} + \dots$$

$$= \frac{i}{p^2-\underset{\text{physical mass}}{m^2}\underbrace{-\Sigma(p)+\delta m^2}_{0}+i\varepsilon}$$

$$\delta m^2 = \Sigma(m^2)$$

$\Longrightarrow$ all predictions use physical mass
- drop external self energies

General procedure

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4}\phi^4$$

1) $\phi_r \equiv Z_\phi^{-1/2}\phi$ or $\phi = Z_\phi^{1/2}\phi_r$

$$\mathcal{L} = Z_\phi \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{m_0^2 Z_\phi}{2}\phi_r^2 - \frac{\lambda_0 Z_\phi^2}{4}\phi_r^4$$

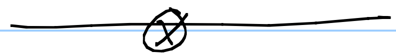
2) use correct mass + coupling constant $\Rightarrow$ counterterms

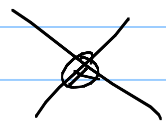
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{m^2}{2}\phi_r^2 - \frac{\lambda}{4}\phi_r^4 + \mathcal{L}_{ct}$$

$$\mathcal{L}_{ct} = \frac{1}{2}(\delta Z)(\partial_\mu \phi_r)^2 + \frac{\delta m^2}{2}\phi_r^2 - \frac{\delta\lambda}{4}\phi_r^4$$

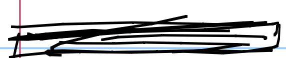
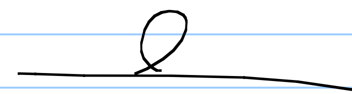
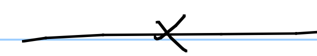
$$\delta Z = Z_\phi - 1$$

3) $\mathcal{L}_{ct}$ as perturbation

 $+ \, i\left(\delta m^2 + \delta Z(p^2 - m^2)\right)$

 $= -6i\,\delta\lambda$

4) Renormalize propagator

 = +  +  + ...

$$= \frac{i}{p^2 - m^2 - \Sigma(p) + \left(\delta m^2 + \delta Z(p^2 - m^2)\right)}$$

$$\Sigma(m^2) + (p^2 - m^2)\,\Sigma'(m^2) + \mathcal{O}\left((p^2 - m^2)^2\right)$$

right mass + norm if $\delta m^2 = \Sigma(m^2)$

$$\delta Z = (Z_\phi - 1) = \Sigma'(m^2) = \frac{\partial}{\partial p^2}\Sigma(p^2)\Big|_{p^2=m^2}$$

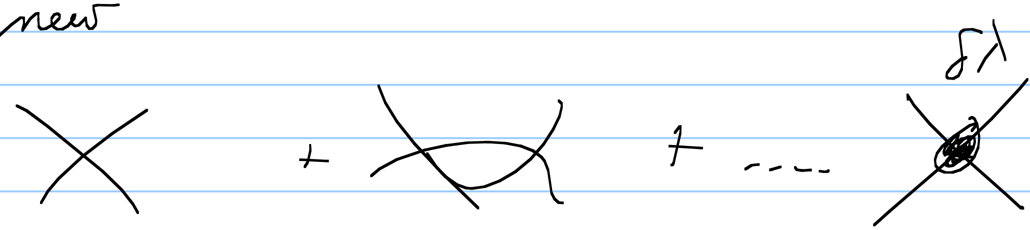
5) Coupling constant

$$\delta m^2 = Z_\phi(m^2 - m_0^2)$$

$$\delta\lambda = \lambda_0 Z_\phi^2 - \lambda$$

new

$-i\mathcal{M}\Big|_{renorm} =$



$= -6i\lambda$, at the renorm point $\Rightarrow \delta\lambda$

2 Procedures

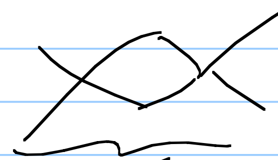
1) BPH - counterterms

2) Conventional or direct

- work with m_0, λ_0 ---

- new rule $Z_\phi^{1/2}$ for each external state

$$\lambda_r = \lambda_0 Z_\phi^4 +$$



$\delta\lambda$

"Renormalizeable" vs "Nonrenormalizeable" theories

Any theory can be renormalized (caveats)

- absorb divergence in parameter of $\mathcal{L}$
- make physical predictions using measured parameters

Caveat: "Whatever is not forbidden is required"

- can't drop terms without reason

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\phi^4 \qquad \text{(no mass term)}$$

 $= \Sigma(p) \Longrightarrow \delta m^2$ required for renorm

Not correct

but $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}(\partial_\mu \phi\, \partial^\mu \phi)^2 + \dots (\partial_\mu \phi\, \partial^\mu \phi)^4 + \dots$ is OK to drop mass

Shift Symmetry $\phi \to \phi + c$

$m^2 \phi^2$ violates symmetry

"Renormalizeable" is separate classification

{ infinities only in a small # of couplings

ie QED – m, e, Z_f, Z_γ

$\lambda\phi$ = m, λ, Z_ϕ

All parameters in $\mathcal{L}$ are dimensionless or positive mass dimensions ✗

e, λ $m^2\phi^2$, $m\bar{\psi}\psi$

Field mass dimensions $[\phi] \sim m^1$, $[\psi] \sim m^{3/2}$, $[A] = m^1$, $[\mathcal{L}] = m^4$

$\Rightarrow$ Field + derivative $\underline{\underline{[\mathcal{O}] \leq m^4}}$ ✗

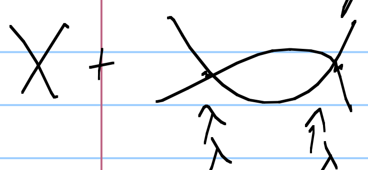
<u>Why?</u> expansion $\left[1 + \#\lambda \frac{1}{d-4} + \dots \right]$ $\leftarrow$ don't generate higher dimensional operators

Example

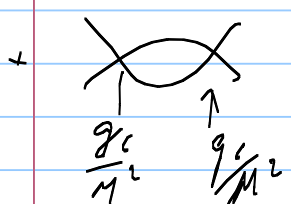
$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - m^2\phi^2 - \frac{\lambda}{4}\phi^4 - \frac{g_6}{M^2}(\partial_\mu\phi)^2\phi^2 - \frac{g_8}{M^4}(\partial_\mu\phi\,\partial^\mu\phi)^2$$

"renorm" "non renormalizable"

One loop



$$= -6i\lambda + (-6i\lambda)^2\left[\frac{1}{d-4} + \cdots\right] = -6i\lambda_{ren}$$



$$+ \frac{g_6^2}{M^4} f(m^4, m^2p^2, p^4)\frac{1}{d-4}$$

ren λ g_6 g_8 $\longleftarrow$ can be renormalized

But need to renormalize $g_8 \sim g_6^2 \frac{1}{d-4}$, g_{10} , g_{12} ---

$\Rightarrow$ more and more operators