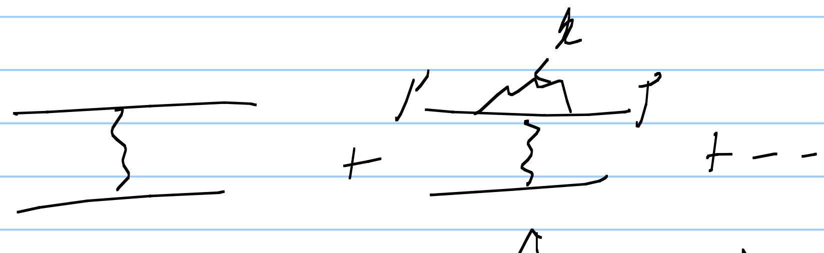
QED 3

Note Title 4/15/2010

"Soft radiation" Infrared Divergences

IR ⇒ low Energy



$$-ie\Gamma_\mu = \int \frac{d^4k}{(2\pi)^4}\, -ie(2p+k)^\mu \, \frac{-i}{k^2+i\varepsilon}\, \frac{i}{(p-k)^2-m^2}\, \frac{i}{(p'-k)^2-m^2}\,(-ie)(2p'+k)_\mu$$

$(p+p'+2k)^\mu$

On shell $\quad (p-k)^2 - m^2 = \overset{m^2}{p^2} - 2p\cdot k - k^2 - m^2 \simeq -2p\cdot k$

$$= \int d^4k \, \frac{1}{k^2}\,\frac{1}{p\cdot k}\,\frac{1}{p'\cdot k} \cdots \sim \frac{d^4k}{k^4} \to \infty$$

Diff from renorm . .

Temporarily $\frac{1}{k^2 - \lambda^2}$ ← photon mass

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{2\alpha}{\pi} \ln \frac{m}{\lambda} f(q^2) \right]$$

← ∞

$$f(q^2) = -\frac{1}{3}\frac{q^2}{m^2} \quad \text{small } q^2$$ ← doesn't mess up renorm.

$$= \left(\ln \frac{-q^2}{m^2} - 1 \right) \quad \text{high } q^2$$

Resolution soft radiation

$$\mathcal{M} \simeq \mathcal{M}_0 \frac{1}{\underbrace{(p'-k)^2 - m^2}_{2p' \cdot k}} (2p'-k)^\lambda$$

Divergence here

$$\frac{d\sigma}{d\Omega} = \cdots \int \frac{d^3k}{(2\pi)^3} \underbrace{\frac{1}{2\omega_k}}_{k} \cdots |\underbrace{M}_{\frac{1}{k}}|^2 \sim \int \frac{d^3k}{k^3} \to \infty$$

"Bremsstrahlung"

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{soft}} = \left(\frac{d\sigma}{d\Omega}\right)_0 \frac{2\alpha}{\pi} \ln \frac{k_{max}}{k_{min}} \; f(q^2)$$

Same $f(q^2)$!

Sum of these is well defined:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{scatt}} + \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{rad.}\\ k < \Delta E}} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{2\alpha}{\pi} f(q^2) \ln \frac{m}{\Delta E} + \overset{\frac{11}{24}}{\text{const.}} \right]$$

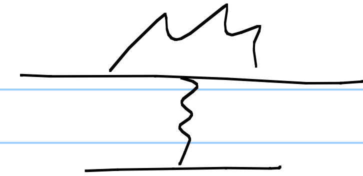
expt resolution

Lamb shift — hidden piece

$F_1(q^2) = 1 + a\, q^2 / \cancel{m^2}$

$\leftarrow \frac{1}{q^2} \leftarrow$

$\leftarrow \delta^3(x) \Longrightarrow$ Lamb shift



$$\mathcal{M} = \frac{1}{q^2} + \text{const}$$

$$a = -\frac{2}{3}\frac{\alpha}{\pi} \ln m^2/\lambda^2 \quad !$$

(~~soft~~ no radiation)

Not on shell $\quad (p-k)^2 - m^2 \sim -2p\cdot k$

$$p^2 - 2p\cdot k + k^2 - m^2$$

Here $E = m - B \Longrightarrow \underset{\uparrow (m-B)^2}{p^2} - 2p\cdot k - m^2 \longrightarrow 2mB$

does not blow up as $k \to 0$

$$a \simeq \frac{2\alpha}{3\pi} \ln\left(\frac{m^2}{mB}\right) \leftarrow$$

g − 2 of electron

$$(i\not{D}+m)(i\not{D}-m)\psi=0 \quad =(-\not{D}\not{D}-m^2)\psi$$

Even though $\not{V}\not{V}=V^2$ but $\not{D}\not{D}\neq D^2$

$$[D_\mu, D_\nu] = [(\partial_\mu - ieA_\mu),(\partial_\nu - ieA_\nu)] = -ieF_{\mu\nu}$$

$$\gamma^\mu\gamma^\nu D_\mu D_\nu = \frac{1}{2}\Big[\underbrace{\{\gamma^\mu,\gamma^\nu\}}_{2g_{\mu\nu}} + \underbrace{[\gamma^\mu,\gamma^\nu]}_{-i\sigma_{\mu\nu}}\Big] D_\mu D_\nu$$

$$= D^2 - \frac{e}{2}\,\underline{\sigma^{\mu\nu}F_{\mu\nu}}$$

$$\left(D^2 - \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu} - m^2\right)\psi = 0$$

Mag field $\quad A_1 = -\frac{1}{2} B y, \quad A_2 = \frac{1}{2} B x, \quad F_{12} = \partial_1 A_2 - \partial_2 A_1 = B$

$$D_i^2 = \partial_i^2 - ie(\underbrace{\partial_i A_i + A_i \partial_i}_{A_i \partial_i}) + A^2$$

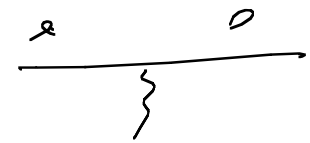
$$= \partial_i^2 - \frac{2ie}{2} B\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = \partial_i^2 - e\vec{B}\cdot(\underbrace{\vec{r}\times\vec{p}}_{L})$$

$$\sigma^{ij} = \epsilon^{ijk}\begin{pmatrix}\sigma^k & 0 \\ 0 & \sigma^k\end{pmatrix} = \epsilon^{ijk}\, 2\vec{S}_k$$

$$\sigma^{12}F_{12} + \sigma^{21}F_{21} = 4\vec{S}\cdot\vec{B}$$

Overall

$$\left[\partial_0^2 + m^2 - \nabla^2 - e\vec{B}\cdot\overset{*}{(\underbrace{\overrightarrow{L+2S}}_{g=2})}\right]\psi = 0$$



Gordon decomposition

$$\bar u(p')\,\gamma_\mu\, u(p) = \bar u(p')\left[\frac{(p+p')^\mu}{2m} + i\,\frac{\sigma^{\mu\nu} q_\nu}{2m}\right]u(p)$$

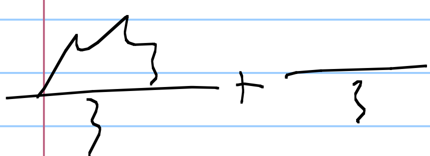
$\not p'/m \qquad \not p/m$

"convective" just like scalar

$\vec L \qquad \vec S$

To set up calc.

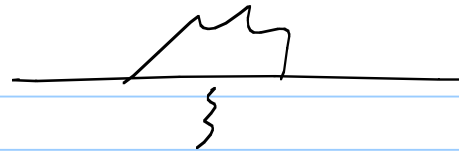
charge renorm Ward ID $\qquad F_1(0)=1$



$$= -ie\,\bar u\left[\gamma_\mu F_1(q^2) + i\,\frac{\sigma^{\mu\nu}}{2m} q_\nu F_2(q^2)\right]u(p)$$

$$\Rightarrow -ie\,\bar u\left[F_1(q^2)\,\cancel{\gamma}\frac{(p+p')^\mu}{2m} + i\,\frac{\sigma^{\mu\nu}}{2m} q_\nu\left[F_1(q^2)+F_2(q^2)\right]\right]u$$

$$\mu = \frac{e}{2m}\left[1 + F_2(0)\right] \Rightarrow \underline{\underline{[g-2] = F_2(0)}}$$

To calculate  + pick out $F_2(q^2=0)$

$$F_2(q^2) = \frac{\alpha}{2\pi}\int_0^1 dx \, \frac{m_e^2}{m_e^2 - q^2(1-x)x} \xrightarrow{q^2\to 0} \frac{\alpha}{2\pi} \quad ✓$$

↑ Schwinger

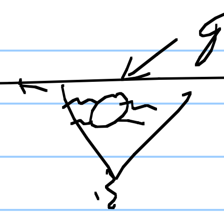
$\Longrightarrow$

th. $a_e = \frac{1}{2}(g-2) = 1,159,652,175.7\ (8.5) \times 10^{-12}$

↑ $\alpha = ?$

$$a_e(\text{expt}) - a_e(\text{th}) = 12.6\ (9.5) \times 10^{-12}$$

Muon – less clean – more sensitive to quark



$$a_\mu(\text{exp}) - a_\mu(\text{th}) = 209\ (97) \times 10^{-11}$$

↑ ??

$\Rightarrow$ "Yang Mills" - gauge theory $\underline{SU(N)}$

QED - $\psi \to e^{-i\alpha(x)}\psi$ $\qquad U(1)$

$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$

Practical Group theory

2 fields $\qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

$\psi \Rightarrow \underline{\psi' = U\psi}$

2×2 unitary

$$\boxed{U = e^{-iH}}$$

$U = e^{-i[\alpha_0(x) 1 + \vec{\alpha}\cdot\vec{\tau}]}$

U(1) $\qquad$ SU(2) $\qquad \vec{\tau} = \vec{\sigma}$ = Pauli matrices

$SU(2) \rightarrow$ Unitary 2×2 with $\det U = 1$

$$\det U = e^{i\, \mathrm{Tr}(\underbrace{\ln U}_{\vec{\alpha}\cdot\vec{\tau}})}$$

$\uparrow$ traceless

<u>Invariant</u>

$$\mathcal{L} \doteq \bar{\Psi}\, \mathcal{O}\, \Psi \longrightarrow \bar{\Psi}\, U^{\dagger}\, \mathcal{O}\, U \Psi = \bar{\Psi}\, \mathcal{O}\, \Psi \quad \text{if } \mathcal{O} = \underset{\substack{2\times 2 \\ \text{unit}}}{\mathbb{1}}$$

$$\mathcal{O} = (i\not{\partial} - \mathrm{m}) \qquad \checkmark$$

$$\uparrow\ \mathrm{m} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Look at $\bar{\psi}\,\tau^i\,\psi \Rightarrow \bar{\psi}\,\underbrace{U^+\tau^i U}_{\substack{2\times 2 \\ \text{Hermetian} \\ \text{Traceless}}}\psi = \underbrace{\bar{\psi}\,\tau^j\,\psi}\,R^{ij}$

2×2 Hermetian, Traceless $\Big\}\ \propto \tau^j$

$$\bar{\psi}\,\tau^i\,\psi \rightarrow \psi'\,\tau^i\,\psi' = R^{ij}\,\bar{\psi}\,\tau^j\,\psi$$

$$U^+\tau^i U = R^{ij}\tau^j \Rightarrow R^{ij} = \tfrac{1}{2}\,\mathrm{Tr}\left(U^+\tau^i U\,\tau^j\right)$$

R^{ij}: Rotation matrix, $O(3)$ ~

U: $SU(2)$

Invariants again

$$\mathcal{L} = \bar{\Psi}' \tau^i \psi \quad \pi^i$$

↑ Transform like vector

$$= \bar{\Psi}' (\vec{\tau} \cdot \vec{\pi}) \psi$$

If $\pi \to \pi'$ such that $(\vec{\tau} \cdot \vec{\pi}') = U (\vec{\tau} \cdot \vec{\pi}) U^\dagger$ ⟵ ★★

then $\mathcal{L} \to \bar{\Psi} \underbrace{U^\dagger U}_{} (\vec{\tau} \cdot \vec{\pi}) \underbrace{U^\dagger U}_{} \psi = \bar{\Psi} \vec{\tau} \cdot \vec{\pi} \psi$

Explicit $\mathrm{Tr}(\tau^i (\vec{\tau} \cdot \vec{\pi}')) = 2\pi'^i = \mathrm{Tr}(\tau^i U \tau^j U^\dagger) \pi^j$

$$\pi'^i = R^{ij} \pi^j \quad \checkmark$$