Introducing the Fields 2

Note Title 2/2/2010

Propagator

$$i D_F(x-x') = \langle 0 | T(\phi(x,t)\,\phi(x',t')) | 0 \rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega} \left[\theta(t-t')\, e^{-ip\cdot(x-x')} + \theta(t'-t)\, e^{-ip\cdot(x'-x)} \right] \qquad \omega = \sqrt{\vec{p}^2+m^2}$$

$$= \int \frac{d^4q}{(2\pi)^4} \frac{i\, e^{-iq\cdot(x-x')}}{q^2-m^2+i\varepsilon} \qquad \varepsilon = 0^+$$

$$d^4q = dq_0\, d^3q \qquad q^2 = q_0^2 - \vec{q}^{\,2}$$

$$\int_{-\infty}^{\infty} dq_0 \frac{e^{-iq_0(t-t')}}{q_0^2-\vec{q}^{\,2}-m^2+i\varepsilon} = \oint_{\substack{LHP \\ (UHP)}} dq_0 \frac{e^{-iq_0(t-t')}}{q_0^2 - \dots} = -2\pi i \,\mathrm{Res}(q_0 = \omega_q - i\varepsilon)$$

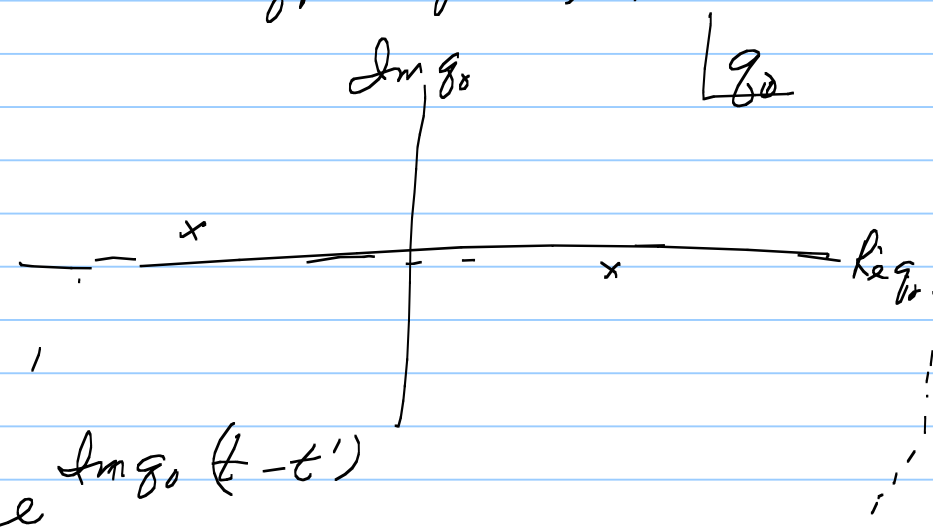
$i\varepsilon$ Physics

Poles $\quad q_0^2 - \vec{q}^{\,2} - m^2 + i\varepsilon = 0 \qquad\qquad q_0^2 = (\vec{q}^{\,2} + m^2) - i\varepsilon$

$$q_0 = \sqrt{\vec{q}^{\,2} + m^2} - i\varepsilon$$

$$q_0 = -\sqrt{\vec{q}^{\,2} + m^2} + i\varepsilon$$

$$\omega_q = \sqrt{\vec{q}^{\,2} + m^2}$$



To do contour integral

$$e^{-iq_0(t-t')} = e^{-i\,\mathrm{Re}\,q_0\,(t-t')}\; e^{\mathrm{Im}\,q_0\,(t-t')}$$

if $t > t'$, $t - t' > 0$ $\quad \mathrm{Im}\, q_0 < 0$ on circle at $\infty \Rightarrow e^{-\infty} \Rightarrow$ LHP

if $t' > t$, $t - t' < 0$ $\quad \mathrm{Im}\, q_0 > 0$ $\quad$ " $\quad$ " $\quad \Rightarrow$ UHP

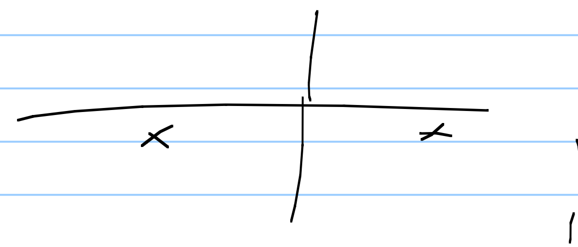
$$\frac{1}{q^2 - m^2 + i\varepsilon} = \frac{1}{q_0 - \omega_q + i\varepsilon}\;\frac{1}{q_0 + \omega_q - i\varepsilon}$$

$$i D_F = -\cancel{2\pi i}\,\cancel{i}\int \frac{d^3q}{(2\pi)^{\cancel{4}\,3}}\left[\theta(t-t')\frac{1}{2\omega_q}\,e^{-i\omega_q(t-t')}\,e^{+i\vec{q}\cdot(\vec{x}-\vec{x}')} - \theta(t'-t)\frac{1}{-2\omega_q}\,e^{-i(-\omega_q)(t-t')}\,e^{i\vec{q}\cdot(\vec{x}-\vec{x}')}\right]$$

$$= \int\frac{d^3q}{(2\pi)^3}\,\frac{1}{2\omega_q}\left[\theta(t-t')\,e^{-iq\cdot(x-x')} + \theta(t'-t)\,e^{+iq\cdot(x-x')}\right] \checkmark$$

Recall E+M Green functions for radiation (retarded G.F.)

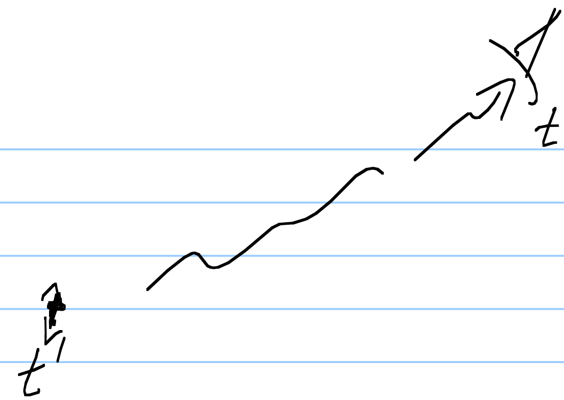
$$\frac{e^{-iq\cdot(x-x')}}{(q_0+i\varepsilon) - \vec{q}^{\,2}}$$



$t > t'$ only

Physics

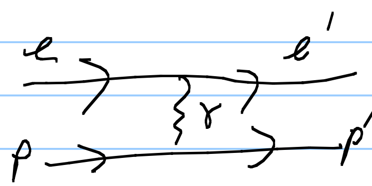
E + M



Want $t > t'$ only for causality

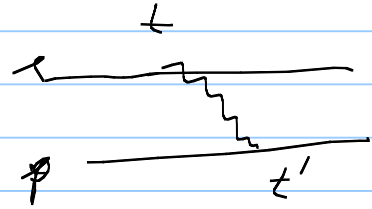
QFT

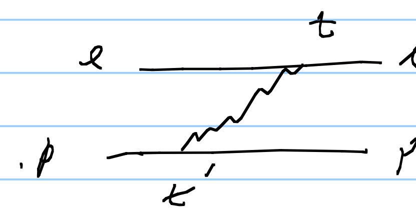
Interactions



$\leftarrow$ Feynman diagrams (no time implied)

Physics is





(time ordered pictures)

Pert theory $\displaystyle\sum_I{}' \frac{\langle f|V|I\rangle\langle I|V|i\rangle}{E - E_I}$ $\Rightarrow$ 2 intermediate states

Zero point energy

$$\sum_s \frac{1}{2}\hbar\omega_s\left(a(s)\,a^+(s) + a^+(s)\,a(s)\right) = \sum_s \hbar\omega\, a^+(s)\,a(s) + E_0$$

$$E_0 = \sum_s \frac{1}{2}\hbar\omega_s = \underbrace{V\int\frac{d^3p}{(2\pi\hbar)^3}}_{\delta(0)}\ \frac{1}{2}\hbar\omega_p \rightarrow \infty$$

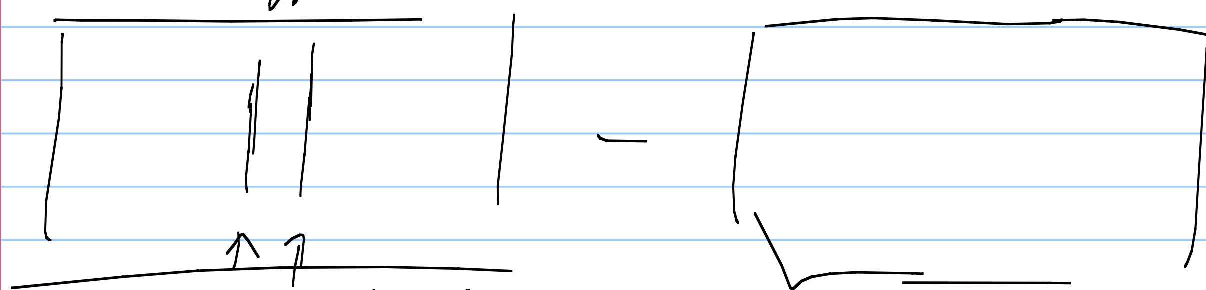
Various possibilities

1) really not infinite — "only N springs" — phonons, strings ...
2) cancellation — bosons $E_0 > 0$, fermion $E_0 < 0$ } "Supersymmetry" $E_0 \Rightarrow 0$
3) gravity finite?
4) ?

Once finite $\Rightarrow$ cosmological constant $\quad \Lambda =$ energy density of empty space

if CMB are $\Lambda \Rightarrow \Lambda = (10^{-3}\,eV)^4$

Casimir effect



$$\Delta E_0 = \sum \tfrac{1}{2}\hbar\omega_{p} - \sum \tfrac{1}{2}\hbar\omega_{no} \neq 0 \qquad \frac{F}{A} = -\frac{\pi^2 \hbar c}{240\, d^4}$$

ω_p: plates; ω_{no}: no plates

Complex scalar field ϕ, ϕ^* $\Rightarrow$ 2 D.O.F.

$$\mathcal{L} = \underline{\partial_\mu} \phi^* \partial^\mu \phi - V(\phi^* \phi)$$

$$V(\phi^* \phi) = \underline{m}^2 \underline{\phi}^* \phi + \underline{\lambda} (\phi^* \phi)^2$$

Comments

1) ϕ, ϕ^* equiv to 2 real fields

$$\phi = \frac{1}{\sqrt{2}} (\phi_1 + i \phi_2)$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2) - \frac{m^2}{2} (\phi_1^2 + \phi_2^2) - \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2$$

2) Symmetry $\left.\begin{aligned} \phi &\to e^{-i\theta} \phi \\ \phi^* &\to e^{+i\theta} \phi^* \end{aligned}\right\}$ phase "U(1)"

3) Eq of motion treat ϕ & ϕ^* as independent

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^*} - \frac{\partial \mathcal{L}}{\partial \phi^*} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^*} = \partial_\mu \phi \quad , \quad \frac{\partial \mathcal{L}}{\partial \phi^*} = -m^2 \phi - 2\lambda(\phi^*\phi)\phi$$

$$\Box \phi - \left(-m^2 \phi - 2\lambda(\phi^*\phi)\phi\right) = 0$$

$$\underbrace{(\Box + m^2)\phi}_{\star} = \underbrace{-2\lambda(\phi^*\phi)}_{\text{interaction}}\phi$$

Quantization

$$\phi(x,t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left[e^{-ip\cdot x}\, a(p) + e^{+ip\cdot x}\, b^\dagger(p) \right]$$

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \partial_0 \phi^* \qquad , \qquad \pi^*(x) = \frac{\partial \mathcal{L}}{\partial \dot\phi^*} = \partial_0 \phi$$

$$\mathcal{H} = \pi \dot\phi + \pi^* \dot\phi^* - \mathcal{L} = \partial_0\phi^* \partial_0\phi + (\vec\nabla\phi^*)(\vec\nabla\phi) + m^2 \phi^*\phi \quad \underline{\underline{\geq 0\,!}}$$

$$[\phi(x,t), \pi(x',t)] = i\,\delta^3(\vec{x}-\vec{x}') \qquad , \qquad [\phi^*, \pi^*] = i\,\delta^3(\quad)$$

if $[a(p), a^\dagger(p')] = (2\pi)^3 \delta^3(p-p')$
$[b(p), b^\dagger(p')] =$ " $\Big\} \Longrightarrow$ commutation rules ✓

✓ $$H = \int d^3x\, \mathcal{H} = \int \frac{d^3p}{(2\pi)^3}\, \omega_p \left(a^\dagger(p)\, a(p) + b^\dagger(p)\, b(p) \right) + \underbrace{E_0}_{2\times \text{real } E_0}$$

Conserved current

$J^\mu = (\rho, \vec{J}) \Rightarrow \partial_\mu J^\mu = 0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}$

$$J^\mu = i\phi^* \partial^\mu \phi - i(\partial^\mu \phi^*)\phi \equiv i\phi^* \overleftrightarrow{\partial_\mu} \phi$$

Check

$$\partial_\mu J^\mu = i\left[\cancel{\partial_\mu \phi^* \partial^\mu \phi} + \underbrace{\phi^* \Box \phi}_{m^2 \phi^* \phi} - \underbrace{(\Box \phi^*)\phi}_{m^2 \phi^* \phi} - \cancel{\partial_\mu \phi^* \partial^\mu \phi}\right] = 0$$

Like E+M conserved charge

$$Q = \int d^3x \, J^0(x,t)$$

$\uparrow \rho$

surface at ∞

$$\frac{dQ}{dt} = \int d^3x \frac{\partial}{\partial t} J^0 = \int d^3x (-\vec{\nabla} \cdot \vec{J}) = -\oint d\vec{A} \cdot \vec{J} \rightarrow 0$$

Calculate Q

$$Q = \int \frac{d^3p}{(2\pi)^3} \left(a^\dagger(p)\, a(p) - b^\dagger(p)\, b(p) \right)$$ *

Interpretation - 2 sets of states

$$|\phi(p)\rangle = a^\dagger(p)\,|0\rangle \quad \leftarrow$$

$$|\bar{\phi}(p)\rangle = b^\dagger(p)\,|0\rangle \quad \leftarrow \qquad \text{antiparticles}$$

$$H\,|\phi(p)\rangle = W_p\,|\phi(p)\rangle \quad , \quad H\,|\bar{\phi}(p)\rangle = W_p\,|\bar{\phi}(p)\rangle$$

$$Q\,|\phi(p)\rangle = +1\,|\phi(p)\rangle \quad , \quad Q\,|\bar{\phi}(p)\rangle = -1\,|\bar{\phi}(p)\rangle$$