Interactions 2

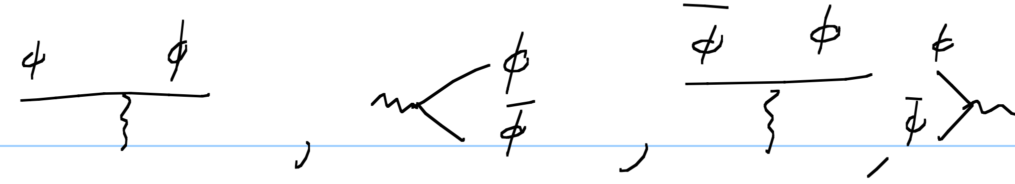
Note Title 2/18/2010

$$i D_\beta (x - x') = \langle \beta | T(\phi(x)\, \phi(x')) | \beta \rangle$$

$\uparrow$

1st order	Order parameter	discontinuous ←
2nd order	" "	continuous

Crossing



$$\langle \phi(p_2) | J_\mu | \phi(p_1) \rangle = \cdots \quad q(p_1+p_2)^\mu \, e^{-i(p_1-p_2)\cdot x}$$

$$\langle \phi(p_2)\, \bar{\phi}(p_1) \, | J_\mu(x) | 0 \rangle = \cdots \quad q(p_2-p_1)^\mu \, e^{+i(p_1+p_2)\cdot x}$$

$$\langle \bar{\phi}(p_1) | J_\mu | \bar{\phi}(p_2) \rangle = \cdots \quad -q(p_1+p_2)^\mu \, e^{-i(p_2-p_1)\cdot x}$$

$$\langle 0 | J_\mu | \phi(p_1)\, \bar{\phi}(p_2) \rangle = \cdots \quad q(p_1-p_2)^\mu \, e^{-i(p_1+p_2)\cdot x}$$

Crossing rules: initial $\Longleftrightarrow$ final state
$p_i \to -p_i$
particle $\Leftrightarrow$ antiparticle

| photons $\varepsilon_\mu \Leftrightarrow \varepsilon^*$
| Dirac $u \Leftrightarrow v$

Dirac

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \left[e^{-ip\cdot x} u(p,s)\, b(p,s) + e^{+ip\cdot x} v(p,s)\, d^\dagger(p,s) \right]$$

$$J_\mu = q \bar{\psi} \gamma_\mu \psi$$

$$\langle f(p_2) | J_\mu | f(p_1) \rangle = q$$

$$= q \langle 0 | \underbrace{b(p_2) \int \frac{d^3p'}{(2\pi)^3} \left[e^{+ip'\cdot x} \bar{u}(p') b^\dagger(p') + \cdots \right]}_{(2\pi)^3 \delta^3(p'-p_2)} \gamma_\mu \int \frac{d^3p}{(2\pi)^3} \left[e^{-ip\cdot x} u(p,s) \underbrace{b(p) + \cdots \right] b^\dagger(p,s)}_{(2\pi)^3 \delta^3(p-p_1)} | 0 \rangle$$

$$= q\, \bar{u}(p_2) \gamma_\mu u(p_1)\, e^{-i(p_1-p_2)\cdot x}$$

$$\langle \bar{f}(p_2) | J_\mu | \bar{f}(p_1) \rangle$$

$$= q \langle 0 | d(p_2) \int \frac{d^3p'}{(2\pi)^3} [+ e^{-ip'\cdot x} \bar{v}(p') d(p')] \gamma_\mu \int \frac{d^3p}{(2\pi)^3} [\cdot \cdots + e^{+ip\cdot x} d^+(p) v(p)] \quad d^+(p_1) | 0 \rangle$$

$$= - q \bar{v}(p_1) \gamma^\mu v(p_2) \quad e^{-i(p_1 - p_2)\cdot x}$$

anti particle has opposite charge

At low velocity $\vec{p}_2 = \vec{p}_1$: $\bar{u} \gamma_\mu u = q(1, \vec{p}/m) \chi^+ \chi = \bar{v} \gamma_\mu v$, with $\chi^+ \chi = 1$

Likewise

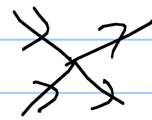
$$\langle f(p_2) \bar{f}(p_1) | J_\mu | 0 \rangle = e^{+i(p_1 + p_2)\cdot x} \quad q \bar{u}(p_2) \gamma_\mu v(p_1)$$

Other interaction

1) $\lambda \phi^4 \Rightarrow \mathcal{L}_I = -\frac{\lambda}{4} \phi^4$

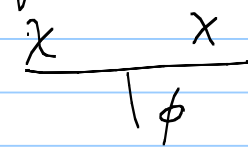
like a $(\text{density})^2$ interaction

$\phi^2 \sim \psi^* \psi \Rightarrow \phi^4 \sim (\psi^* \psi)(\psi^* \psi)$

in scattering 

2) Complex scalar χ, real scalar ϕ $\quad \mathcal{L}_I = -g \chi^\dagger \chi \phi$

– like QED $\quad$ ↑ ~charged particle $\quad$ ↑ $\sim A_\mu$



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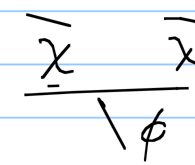
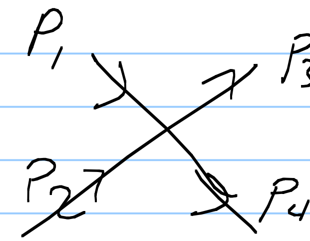


Samples



$$\mathcal{M} = \langle p_3, p_4 \,|\, \frac{\lambda}{4}\phi^4 \,|\, p_1, p_2 \rangle$$

$$= \frac{\lambda}{4} \frac{e^{-i(p_1+p_2-p_3-p_4)\cdot x}}{\sqrt{2W_1\, 2W_2\, 2W_3\, 2W_4}} \times P$$

$\uparrow$ counting factor

To get P

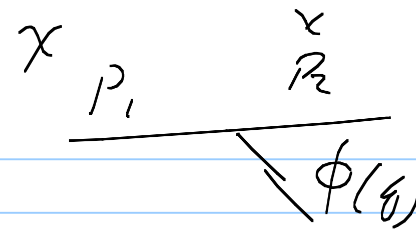
4 ways to annih. p_1
3 way " p_2
2 ways to create p_3
1 way " p_4

$\Bigg\}$ $P = 4!$

$$\mathcal{M} = \left[\frac{e^{-i(\;\;)\cdot x}}{\sqrt{2W_1 \cdots 2W_4}} \right] 6\lambda$$

2) $\mathcal{M} = \langle \chi(p_2)\, \phi(q) \,|\, g\, \chi^\dagger \chi \phi \,|\, \chi(p_1) \rangle$

$= \left[\frac{e^{\cdots}}{\sqrt{\cdots}} \right] g$



Perturbation theory

Plan
1) Time Development operator
2) Calculate transition amplitudes
3) Feynman rules

Recall Interaction Picture $\quad H = H_0 + H_I$

1) Basis states due to H_0
   - for us particle states $|p_i\rangle$
2) Time independent if $H_I = 0$
3) States change due to interactions

$$|\Psi(t)\rangle = U_I(t, t_0)\,|\Psi(t_0)\rangle$$

↑ Time Development Op

$$U_I(t,t_0) = T\, e^{-i\int_{t_0}^{t} dt'\, H_I(t')}$$

Conceptual path

$$i\frac{d}{dt}|\psi_I(t)\rangle = H_I(t)|\psi_I(t)\rangle$$

Integral eq $|\psi_I(t)\rangle - |\psi_I(t_0)\rangle = -i\int dt'\, H_I(t')\,|\psi_I(t')\rangle$

Iterate $|\psi_I(t)\rangle = |\psi_I(t_0)\rangle - i\int dt'\, H_I(t')\left[|\psi_I(t_0)\rangle - i\int_{t_0}^{t'} dt''\, H_I(t'')\,|\psi_I(t'')\rangle\right]$

$$= \left[1 - i\int_{t_0}^{t} dt'\, H_I(t') + (-i)^2\int_{t_0}^{t} dt'\int_{t_0}^{t'} dt''\, H_I(t')H_I(t'') + \dots\right]|\psi_I(t_0)\rangle$$

Change of variables $\int_{t_0}^{t} dt'\int_{t_0}^{t'} dt''\, H(t')H(t'') = \frac{1}{2}\int_{t_0}^{t}\int_{t_0}^{t} dt'\,dt''\, T\big(H(t')H(t'')\big)$

Compact notation $[\dots] = U_I(t,t_0) = T\, e^{-i\int_{t_0}^{t} dt'\, H_I(t')}$

Example $\mathcal{L}_I = -\frac{\lambda}{4}\phi^4$

$t \to -\infty \quad |p_1, p_2\rangle = |\Psi_i\rangle$

Time develop $t \to +\infty \implies U_I(\infty, -\infty)|p_1, p_2\rangle$

Amplitude for finding $|\Psi_f\rangle = |p_3, p_4\rangle$

$$T_{fi} = \langle p_3 p_4 | U_I(\infty, -\infty) | p_1, p_2\rangle$$

↑ S matrix $= T\exp\int_{-\infty}^{+\infty} dt\, H_I(t) = T\exp\left\{-i\int dt\, d^3x\, \mathcal{H}_I(x,t)\right\}$

$$= T\exp\left\{+i\int d^4x\, \mathcal{L}_I(x,t)\right\}$$

our examples $\mathcal{H}_I = -\mathcal{L}_I$

To first order in λ

$$T_{fi} = \langle p_3\, p_4 | \left[1 - i \frac{\lambda}{4} \int d^4x \; \phi^4(x,t) + \cdot \right] | p_1\, p_2 \rangle$$

$$= -i \int d^4x \left(\frac{\lambda}{4} \times 4! \right) \frac{e^{-i(p_1+p_2-p_3-p_4)\cdot x}}{\sqrt{2\omega_1\, 2\omega_2\, 2\omega_3\, 2\omega_4}}$$

$$= -i \; \underbrace{\frac{\delta^4(p_1+p_2-p_3-p_4)}{\sqrt{2\omega_1 \cdots 2\omega_2}}} \quad \mathcal{M}$$

$$\mathcal{M} = 6\lambda \quad \Longrightarrow \quad -i\mathcal{M} = -6i\lambda$$