Interactions 1

Note Title 2/11/10 ~~2/9/2010~~

E + M

Covariant der. $\partial_\mu \to D_\mu = \underline{\partial_\mu + iqA_\mu}$

Scalar

$$\mathcal{L} = (D_\mu\phi)^*(D_\mu\phi) - V(\phi) \qquad \leftarrow \text{complex scalar}$$

Sch.

$$\mathcal{L} = \frac{i}{2}\left(\psi^*(D_0\psi) - (D_0\psi)^*\psi\right) - \frac{(\vec{D}\psi)^*(\vec{D}\psi)}{2m}$$

Dirac

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi$$

Gauge invariance

$$\begin{cases} A_\mu \to A_\mu' = A_\mu + \partial_\mu \Lambda(x,t) \\ \phi \to \phi' = \underline{e^{-iq\Lambda(x,t)}}\,\phi \end{cases}$$

(or ψ

$F_{\mu\nu}' = F_{\mu\nu}$

$\phi^* \phi$ invariant

$$D_\mu \phi \to D_\mu' \phi' = \left[\partial_\mu + iq(A_\mu + \partial_\mu \Lambda)\right] e^{-iq\Lambda}\phi$$

$$= e^{-iq\Lambda}\left[\partial_\mu - iq(\partial_\mu \Lambda) + iq(A_\mu + \partial_\mu \Lambda)\right]\phi$$

$$= e^{-iq\Lambda} D_\mu \phi$$

$\Rightarrow \phi^* D_\mu \phi$ invariant

$(D_\mu \phi)^* (D_\mu \phi)$ invariant

EM Current

$(\rho, \vec{J})$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu J^\mu$$

Maxwell Eq.

$$\frac{\partial \mathcal{L}}{\partial \partial_\mu A_\nu} = -F^{\mu\nu} \quad , \quad \frac{\partial \mathcal{L}}{\partial A_\nu} = -J^\nu \quad \Rightarrow \quad \boxed{\partial_\mu F^{\mu\nu} = J^\nu}$$

Scalar

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underbrace{(D_\mu \phi)^* (D^\mu \phi)}_{-A_\mu J^\mu \; + q^2 A_\mu A^\mu \phi^* \phi} - V(\phi^* \phi)$$

$$(D_\mu \phi)^* (D^\mu \phi) = \left[(\partial_\mu - i q A_\mu) \phi^*\right](\partial^\mu + i q A^\mu)\phi = \partial_\mu \phi^* \partial^\mu \phi - i q A_\mu \left(\phi^* \overrightarrow{\partial^\mu} \phi - \phi^* \overleftarrow{\partial^\mu} \phi\right)$$

$$\underline{+ q^2 A_\mu A^\mu \phi^* \phi}$$

$$J^\mu_{em} = iq\left(\phi^* \partial_\mu \phi - (\partial_\mu \phi^*)\phi\right) \qquad *$$

$$\uparrow = q * J^\mu \leftarrow \text{particle current}$$

$$Q_{em} = \int d^3x\, J^0_{em} = \int \frac{d^3p}{(2\pi)^3}\, q\left(a^\dagger(p)\,a(p) - b^\dagger(p)\,b(p)\right)$$

$$\partial_\mu J^\mu_{em} = 0 \quad \text{(shown previously)}$$

Non rel

$$\xrightarrow{\partial_0 + iqA_0}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2}\left(\psi^* D_0 \psi - (D_0 \psi^*) \psi\right) - \frac{(\vec{D}\psi)^*(\vec{D}\psi)}{2m}$$

$$= \text{free field} - q A_0 \psi^* \psi - iq\vec{A}\, \frac{\left((\vec{\nabla}\psi^*)\psi - \psi^* \vec{\nabla}\psi\right)}{2m} + \underset{\sim}{q\vec{A}\vec{A}\psi^*\psi}$$

$$* \quad J^\mu_{em} = q\left(\psi^*\psi, \frac{1}{2m}\bar{\psi}(-i\vec{\nabla} + i\overleftarrow{\nabla})\psi\right) = \underbrace{q\left(\psi^*\psi, \bar{\psi}\frac{\vec{p}}{m}\psi\right)}_{**}$$

$$H = \int d^3x\, \psi^* \left[\frac{D^2}{2m} + q \overset{\phi}{A_0}\right] \psi \qquad \checkmark$$

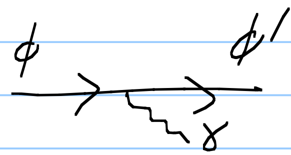
Dirac $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi \quad \Longrightarrow \quad J^\mu_{em} = \underline{q \bar{\psi} \gamma^\mu \psi}$

What does $A_\mu J^\mu$ do?

$$J_\mu A^\mu = q\left[\phi^* i\partial_\mu \phi - (i\partial_\mu \phi^*)\phi\right] A^\mu \qquad \longleftarrow \quad iq\,\phi^* \overleftrightarrow{\partial_\mu}\,\phi\, A^\mu$$

ϕ^*: $a^\dagger, b$ $\qquad$ ϕ: $a + b^\dagger$ $\qquad$ A^μ: $a_\gamma + a_\gamma^\dagger$

Transition



$+ \;-\; -$

Current

$$\langle \phi(p_2) \,|\, J_\mu \,|\, \phi(p_1)\rangle$$

$$q(p_1 + p_2)_\mu$$

$$= \langle 0|\, a(p_2) \int \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2w}}\left[e^{+ip'\cdot x} a^\dagger(p') + \cdots\right] iq\overleftrightarrow{\partial_\mu} \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2w}}\left[e^{-ip\cdot x} a(p) + \cdots\right] a^\dagger(p_1)\,|0\rangle$$

$$\underbrace{[a(p), a^\dagger(p_1)]}_{(2\pi)^3\delta^3(p - p_1)} + a^\dagger(p_1)\, a(p)|0\rangle \to 0$$

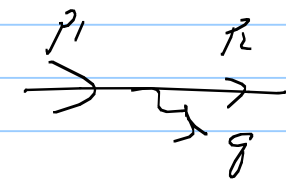
$$= \frac{1}{\sqrt{2\omega_1 \, 2\omega_2}} \; q \, (p_1 + p_2)_\mu \; e^{-i(p_1 - p_2)\cdot x}$$

$\vec{v}$

Note as $p_1 \to p_2$ $= \langle J_\mu \rangle = q \dfrac{P_\mu}{E} = q\left(1, \dfrac{\vec{p}}{E}\right)$

$$\underline{\mathcal{L}_{int} = -J_\mu A^\mu}$$

$-J_\mu A^\mu$ $\quad \int \frac{d^3 q'}{(2\pi)^3} \left[\cdots + e^{i q' \cdot x} a^\dagger(q') \, \varepsilon^*(q')^\mu \right] \frac{1}{\sqrt{2\omega_q}}$

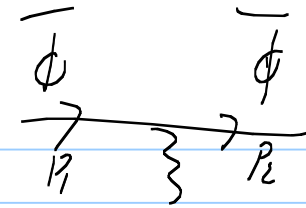


$$\text{Amp} = \langle \phi(p_2) \, A(q) \,|\, \mathcal{L}_{int} \,|\, \phi(p_1) \rangle$$

$$= \frac{1}{\sqrt{2\omega_1 \, 2\omega_2}} \, \frac{1}{\sqrt{2\omega_q}} \; q\,(p_1 + p_2)_\mu \; \varepsilon^{*\mu} \; e^{-i(p_1 - p_2 - q)\cdot x}$$

Q_{elec}

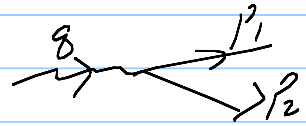
$$\langle \bar{\Phi}(p_2) \,|\, J_\mu \,|\, \bar{\Phi}(p_1) \rangle \qquad -q(p_1+p_2)_\mu$$



$$= \langle 0|\, b(p_2) \int \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2\omega'}} \left[\cdots + e^{-ip'\cdot x}\, b(p') \right] iq \overleftrightarrow{\partial}_\mu \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} \left[\cdots + e^{+ip\cdot x}\, b^\dagger(p) \right] b^\dagger(p_1) |0\rangle$$

$$= \frac{-q}{\sqrt{2\omega_1 2\omega_2}} (p_1+p_2)_\mu \, e^{-i(p_1-p_2)x}$$

One more



$$\langle \bar{\phi}(p_2)\, \phi(p_1) \,|\, J_\mu A^\mu \,|\, \gamma(q) \rangle$$

$$= \frac{1}{\sqrt{2\omega_1 2\omega_2}} \frac{1}{\sqrt{2\omega_q}} \, e^{-i(q-p_1-p_2)\cdot x} \quad q(p_1-p_2)^\mu \cdot \epsilon_\mu$$

Rules

1) initial $e^{-ip\cdot x}$

final $e^{+ip\cdot x}$

2) $\frac{1}{\sqrt{2\omega}}$ for any boson (1 for ~~for~~ fermion)

3) ε_μ for initial photon , $\varepsilon^{*\mu}$ for final

4 $\partial_\mu \to -ip_\mu$ for initial state

$+ip_\mu$ final state

5) Currents $q_0 \Rightarrow +q$ ~~for~~ particle

$-q$ for anti

6) Dirac (next)

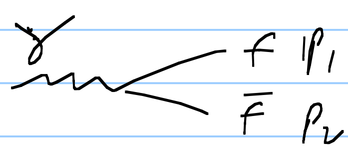
$u(p)$ for incoming particle

$\bar{u}(p)$ for final particle

$\bar{v}(p)$ for $\underline{\underline{\text{incoming}}}$ antiparticle $\Big\}$!

$v(p)$ for $\underline{\underline{\text{outgoing}}}$ antiparticle

Ex



$$\text{Amp} = \frac{1}{\sqrt{2 w_q}} \; 1 \times 1 \; e^{-i(q - p_1 - p_2)\cdot x} \; q \, \varepsilon_\mu(q) \, \bar{u}(p_1) \gamma^\mu v(p_2)$$

outgoing f

$\bar{f}$ out