Gravity 2

Note Title 12/10/2009

Yang Mills		Gravity
$\psi \to \psi' = e^{i\vec{\alpha}(x)\cdot\frac{\vec{\tau}}{2}}\psi = U\psi$	Invariance	$\psi' = e^{i\frac{w^{ab}S_{ab}}{2}}\psi$; $S_{ab} = \frac{i\sigma_{ab}}{2}$
$A_\mu = \frac{A^i\cdot\tau^i}{2}$	New Fields	$A_\mu = \frac{A^{ab}_\mu S_{ab}}{2}$, V^μ_a
$D_\mu = \partial_\mu - igA_\mu$	Cov. Derivative	$D_\mu = \partial_\mu + A_\mu$; $S(\Lambda) = e^{-i\frac{\omega_{ab}\sigma^{ab}}{4}}$
$A'_\mu = UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}$	Transf.	$A_\mu = SA_\mu S - [\partial_\mu S]S^{-1}$
$[D_\mu, D_\nu] = -ig\frac{\tau^i}{2}F^i_{\mu\nu}$	Field Strength	$[D_\mu, D_\nu] = \frac{S_{ab}R^{ab}_{\mu\nu}}{2}$

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ Lagrangian $\mathcal{L} = \frac{2}{\kappa^2} \underbrace{V^\mu_c V^\nu_b R^{ab}_{\mu\nu}}_{R}$

$+ c R_{ab}^{\mu\nu} R^{ab}_{\mu\nu}$

$\Longrightarrow$ First order Formalism

2 fields V^μ_a , R^{ab}_μ

Connected by eq of motion $\underline{A^{ab}_\mu} = g^{\rho\sigma} V^a_\rho \frac{\partial}{\partial x^\mu} V^b_\sigma$

Second Order — only $g_{\mu\nu}$

Vector field $A^\mu \Rightarrow A'^\mu = \Lambda^\mu{}_\nu(x) A^\nu$

Covariant derivatives

$$D_\mu A^\lambda = \partial_\mu A^\lambda + \Gamma^\lambda_{\mu\nu} A^\nu$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}\left[\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right]$$

affine connection
– gauge field

$$[D_\mu, D_\nu] A_\alpha = \underbrace{R^\beta{}_{\alpha\mu\nu}}_{\text{curvature}} A_\beta$$

$$R^\beta{}_{\alpha\mu\nu} = \partial_\mu \Gamma^\beta_{\alpha\nu} - \partial_\nu \Gamma^\beta_{\alpha\mu} + \Gamma^\lambda_{\alpha\nu} \Gamma^\beta_{\lambda\mu} - \Gamma^\lambda_{\alpha\mu} \Gamma^\beta_{\lambda\nu}$$

$$\mathcal{L} = \sqrt{g}\left\{ \Lambda + \frac{2}{\kappa^2} R + \underbrace{c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu}}_{\text{Postpone}} + \cdots \right\}$$

Derivative expansion $\quad \Gamma \sim \partial_\mu g$

$$R \sim \partial\Gamma + \Gamma^2 \sim \partial^2 g$$

Get

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \underbrace{8\pi G \Lambda}_{} = -8\pi G\, T_{\mu\nu} \qquad \kappa^2 = 32\pi G$$

$$\lambda = -8\pi G \Lambda$$

$\uparrow$ $10^{-56}\,\text{cm}^{-2} \leftarrow 10^{-46}\,\text{GeV}^4$

Drop

$$g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}(x)$$

$$g_{\mu\nu} = \eta_{\mu\nu} - \kappa h_{\mu\nu} + \frac{\kappa^2}{2} h^{\lambda}{}_{\mu} h_{\lambda\nu} + \cdots$$

$$R = \underbrace{\kappa\left[\Box h^{\lambda}{}_{\lambda} - \partial_\mu \partial_\nu h^{\mu\nu}\right]} + \underbrace{\mathcal{O}(h^2)}_{H.W.} \quad \cancel{+}\,\mathcal{O}(h^3)$$

$$R_{\mu\nu} = \kappa\left[\partial_\mu \partial_\nu h^{\lambda}{}_{\lambda} + \partial_\lambda \partial^\lambda h_{\mu\nu} - \partial_\mu \partial_\lambda h^{\lambda}{}_{\nu} - \partial_\nu \partial_\lambda h^{\lambda}{}_{\mu}\right] + \cdots$$

Gauge choice $\partial^\lambda h_{\mu\lambda} = \frac{1}{2}\partial_\mu h^{\lambda}{}_{\lambda}$ Harmonic gauge

$$\underbrace{\Box h_{\mu\nu}} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^{\lambda}{}_{\lambda}\right)$$

Gravity waves
Grav. field

R^2 terms

(Stelle)

$$\mathcal{L} = \frac{2}{\kappa^2} R + c R^2$$

$\Box h$; $(\Box h)^2$

c dimensionless

$\frac{2}{\kappa^2} \sim M_p^2$

$$\Longrightarrow \quad \Box h + \kappa^2 c \,\Box\Box h = 8\pi G T$$

$$G(x) = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq\cdot x}}{q^2 + \kappa^2 c q^4} = \int \ldots \left[\frac{1}{q^2} - \frac{1}{q^2 + 1/\kappa^2 c} \right]$$

$\frac{1}{\kappa^2 c} \sim \frac{M_p^2}{c}$

$$V(r) = -GMm \left[\frac{1}{r} - \frac{e^{-r/\sqrt{\kappa^2 c}}}{r} \right]$$

$$= -\frac{GMm}{r} - 128\pi^2 G^3 Mm\, \delta^3(x)$$

$c \sim 1$, $\underline{\sqrt{\kappa^2 c} = 10^{-35} m}$

Planck distance

Background Field Quantization

– retain symmetries

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}$$

$$\frac{2}{\kappa^2}\sqrt{g}\,R = \sqrt{\bar{g}}\left\{\frac{2}{\kappa^2}\bar{R} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} \cdots \right\}$$

$$\mathcal{L}^{(1)} = \frac{1}{\kappa} h^{\mu\nu}\left[\bar{g}_{\mu\nu}\bar{R} - 2\bar{R}_{\mu\nu}\right] \longrightarrow 0 \quad \text{by eq of motion}$$

$$\mathcal{L}^{(2)} = \frac{1}{2}\left[D_\alpha h_{\mu\nu} D^\alpha h^{\mu\nu} \cdots\cdots \bar{R}^{\mu\nu} h^\lambda_\lambda h_{\mu\nu} \cdot\right]$$

$$= h_{\alpha\beta}\, D^{\alpha\beta\mu\nu}\, h_{\mu\nu}$$

$$\uparrow (D + \Gamma)^2 + \sigma$$

Gauge fixing $\quad G^a = \sqrt[4]{g}\,\left(D^\nu h_{\mu\nu} - \frac{1}{2} D_\mu h^\lambda{}_\lambda\right) V^{\mu a}$

$\uparrow$ vierbein

$$\mathcal{L}_{gf} = \underbrace{G^a G_a}_{h^2} \to D^{\kappa\beta\mu\nu}$$

$$\underbrace{\left|\frac{\partial G_a}{\partial \varepsilon_b}\right|} \leftarrow \text{in P.I.} \qquad \det M = \int d\eta\, d\bar\eta\; e^{i\int d^4x\, \bar\eta M \eta}$$

$$h_{\mu\nu} \to h'_{\mu\nu} + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu$$

$$\frac{\partial G^a}{\partial \varepsilon^b} = \sqrt[4]{g}\left[\delta^{ab} D_\nu D^\nu + [D_b, D_\mu]\, V^{\mu a}\right]$$

Ghost fields η_a

$$\mathcal{L} = \sqrt{\bar{g}}\, \eta^{*\mu} \left[D_\mu D_\nu \bar{g}_{\mu\alpha} - R_{\mu\alpha} \right] \eta^\alpha$$

Then

$$S = \int d^4x \sqrt{\bar{g}} \left[\frac{2}{\kappa^2} R^2 + h_{\alpha\beta} D^{\alpha\beta\mu\nu} h_{\mu\nu} + \eta \left[\quad \right] \eta \right] =$$

Renormalization

Heat kernel $\quad h \, D \, h$

$\qquad\qquad\qquad\quad \uparrow$

't Hooft + Veltman

$$\mathcal{L}^{div} = \frac{1}{8\pi^2 \epsilon}\left[\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right] \quad \leftarrow \text{respects GR.}$$

Renorm

$$\hat{c}_1 = c_1 + \frac{1}{960\pi^2\epsilon}$$

$$\hat{c}_2 = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity is one loop finite, $\quad R_{\mu\nu} = 0 = R$

– only in 4d

$$R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \neq (\ldots R_{\mu\nu} R^{\mu\nu} + \ldots R^2)$$

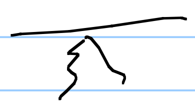
Fails at 2 loops $\quad R^{\mu\nu\alpha\beta} R_{\alpha\beta\gamma\delta} R^{\gamma\delta}{}_{\mu\nu} \neq 0$

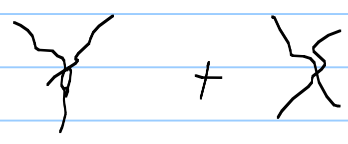
Feynman rules

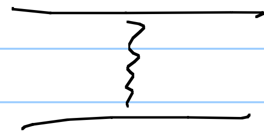
$\mu\nu$  $\alpha\beta$ $\quad = \dfrac{i}{q^2 + i\varepsilon} P_{\mu\nu\alpha\beta} \quad ; \quad P_{\mu\nu\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta} \right]$

$$= \tau^i_{\mu\nu} = -\frac{i\kappa}{2}\left(p_\mu p'_\nu + p'_\mu p_\nu - g_{\mu\nu}[p \cdot p' - m^2] \right)$$

$\leftarrow T^{\mu\nu}$

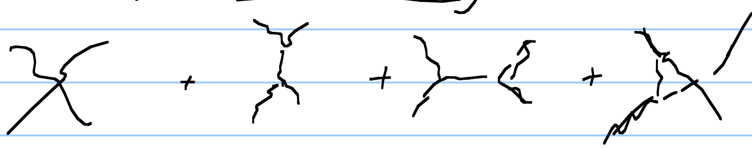




Calculate  $= -i\tilde{T}^{\mu\nu}\frac{i}{q^2}P_{\mu\nu\alpha\beta}\tilde{T}_{\alpha\beta}$

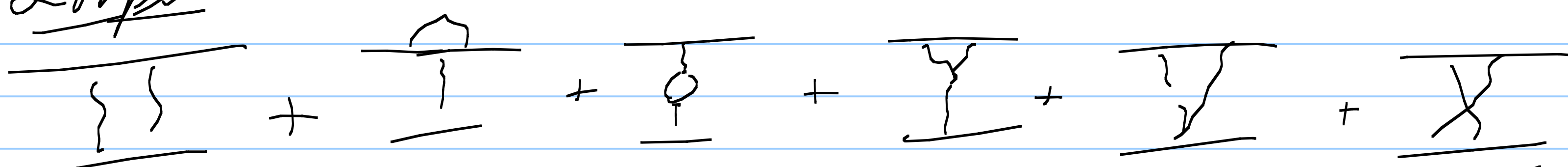
$\overset{NR}{\Rightarrow} 4\pi G m_1 m_2 \frac{1}{q^2} \longrightarrow \frac{G m_1 m_2}{r}$

Grav. Grav. scattering



$\Rightarrow A(++,++) = A(--,--) = \kappa^2 \frac{s^3}{stu}$

Loops



quantum

$$V(r) = \frac{G m_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

classical
Post Newtonian approx.

Energy Expansion

Momentum $\mathcal{M}(q) \approx \mathcal{M}_0 \left[1 + Gq^2 + Gq^2 \ln q^2 + \cdots \right]$

quantum corrections get big
$q^2 = \frac{1}{G} = M_p^2$

$\Rightarrow$ Quantum GR breaks down at Planck scale

Low E theory fine

High E not so fine

QED
QFT