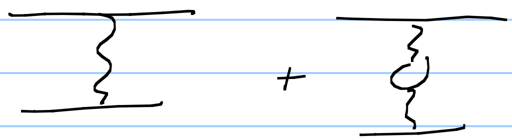
Gauge Theory 3 — Oct 1

Note Title — 10/1/2009

Reminder: Running coupling in QED

$-q^2 >> m^2$, $q^2 = \mu_R^2$



$$+ \dots \Longrightarrow \frac{e_0^2}{q^2}\left(1-\Pi(q^2)\right) \longrightarrow \frac{e_0^2}{q^2\left(1+\Pi(q^2)\right)}$$

Calculation

$$\Pi(q^2) = \frac{e_0^2}{6\pi^2}\left\{ \frac{1}{\underset{4-d}{\varepsilon}} + \ln\sqrt{4\pi} - \frac{\underset{0.577\dots}{\gamma}}{2} + \frac{5}{6} - \frac{1}{2}\ln -q^2/\mu^2 + \dots \right\}$$

Define

$$e^2(\mu_R^2) = e_0^2\left(1-\Pi(q^2=\mu_R^2)\right)$$

$$e^2(q^2) = e^2(\mu_R^2)\left[1 + \frac{e^2}{12\pi^2}\ln\frac{-q^2}{\mu_R^2}\right.$$

$$\beta = \mu_R\frac{\partial e}{\partial \mu_R} = \frac{e^3}{12\pi^2}$$

Trick – divergences determine logs

$-q^2 >> m^2$ (m^2 not important)

μ enters only $\mu^{4-d} \int \frac{d^d k}{(2\pi)^d}$

$$\Rightarrow \mu^{4-d}\left\{\frac{1}{4-d}\right\} = e^{(4-d)\ln\mu}\frac{1}{4-d} = \frac{1}{4-d}\left(1+(4-d)\ln\mu\right)$$

$$= \underbrace{\frac{1}{\varepsilon} + \ln\mu} = \left(\frac{1}{\varepsilon} - \frac{1}{2}\ln\frac{q^2}{\mu^2}\right)$$

Only other scale is q^2

$$\left[\frac{1}{\varepsilon} - \frac{1}{2}\ln q^2/\mu^2\right] = \left(\frac{\mu^2}{-q^2}\right)^{\varepsilon/2}\frac{1}{\varepsilon}$$

Look for $\frac{1}{\varepsilon} \Rightarrow$ know $\ln q^2$

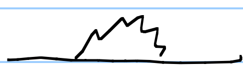
If $g = g_0\left(1 - b_0 g^3\left\{\frac{1}{\varepsilon} + \cdots\right\}\right) \Rightarrow \beta = \mu_R \frac{\partial g}{\partial \mu_R} = b_0 g^3$

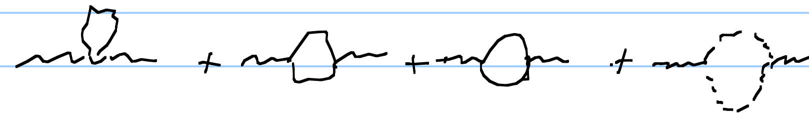
Charge renorm in YM

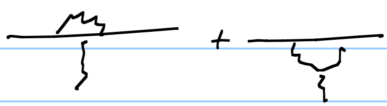
Chen, + Li

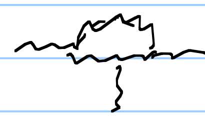
Many ways

Various definitions g_{ren}

 $= \dfrac{Z_2^{-1}}{\not p - m}$ ← × ;

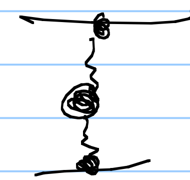
 $\propto \dfrac{Z_3^{-1}}{q^2}$

 $\to Z_{1F}^{-1}\,\gamma_\mu$;

 $\sim Z_{1g}^{-1}\,(\ \ )$;

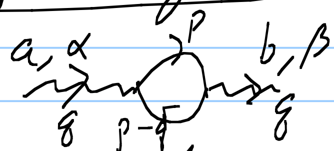
 $+ \cdots \quad Z_4$

$$g_{ren} = Z_{1F}^{-1}\, Z_2\, Z_3^{-1/2}\, g_0$$
$$= Z_{1g}^{-1}\, Z_3^{-1/2}\, g_0$$
$$= Z_4^{-1/2}\, Z_3\, g_0$$
$$= Z_{1\phi}^{-1}\, Z_{2\phi}\, Z_3^{-1/2}\, g_0$$

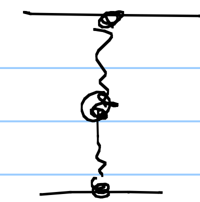


Same Slavnov Taylor identities

Vacuum polarization

Fermion 

↙ fermion loop



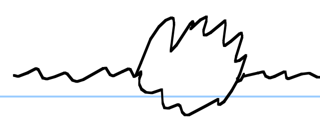
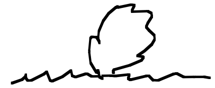
$$i\Pi^{ab}_{\alpha\beta} = (-1)\int \frac{d^4p}{(2\pi)^4}\, \mathrm{Tr}\left[\left(-ig\gamma_\alpha \frac{\lambda^a}{2}\right)\frac{i}{\not p - m + i\epsilon}\left(-ig\gamma_\beta\frac{\lambda^b}{2}\right)\frac{1}{\not p - \not q - m}\right]$$

↖ Dirac + color

Color $\quad \mathrm{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$

$$i\Pi^{ab}_{\alpha\beta} = i\left(q_\alpha q_\beta - g_{\alpha\beta}q^2\right)\left(\frac{\mu^2}{-q^2}\right)^{\epsilon/2}\delta^{ab}\,\frac{g^2}{12\pi^2}\,\frac{1}{\epsilon} + \cdots$$

n_F Types of fermions $\Rightarrow$ mult by n_F

Gluons  + 

(i) (ii)

(ii) = 0 because $\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \to \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2} = 0$

Oddity of dim reg

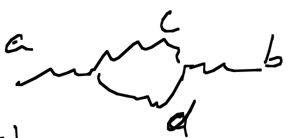
$$\int \frac{d^4 l}{(2\pi)^4}\frac{1}{(l^2+m^2)^n} \to \mu^{4-d}\int\frac{d^dl}{(2\pi)^d}\frac{1}{(l^2+m^2)^n}$$

$$= \frac{i}{16\pi^2}\frac{1}{(4\pi)^{\frac{d-4}{2}}}\Gamma(n-d/2)\left(\frac{\mu}{m}\right)^{4-d} m^{4-2n}$$

Then $m \to 0$

$n=0$ $\int\frac{d^4l}{(2\pi)^4} \to 0$ & $n=1$ $\int\frac{d^4l}{(2\pi)^4}\frac{1}{l^2}\to 0$!

Note $n=3$ $\int \frac{d^4l}{(2\pi)^4}\frac{1}{(l^2)^3}\to\infty$ is physical



i)  $\swarrow$ $\frac{1}{2}$ loop of identical bosons

$$i\Pi^{ab}_{\alpha\beta}(q^2) = \frac{1}{2}(-ig)^2(-i)^2 \int \frac{d^4k}{(2\pi)^4}\, f^{acd} f^{bcd} \frac{N_{\alpha\beta}}{[k^2+i\varepsilon][(k-q)^2+i\varepsilon]}$$

Color factor $f^{acd} f^{bcd} = C_2(\text{adjoint})\,\delta^{ab} = N\delta^{ab}$

$$i\Pi^{ab}_{\alpha\beta} = i\left[\frac{11}{3} q_\alpha q_\beta - \frac{19}{6} g_{\alpha\beta} q^2\right]\left[\frac{-g^2}{16\pi^2} C_2(\text{adj})\,\delta^{ab} \frac{1}{\varepsilon}\left(\frac{\mu^2}{q^2}\right)^{\varepsilon/2} + \cdots\right.$$

not gauge invariant $q^\alpha \Pi_{\alpha\beta} \neq 0$

Practical group theory $C_2(\text{adj})$, $C_2(\text{fund})$

Use $\lambda^a_{ij}\lambda^a_{kl} = 2\left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\right)$

Then $(\lambda^a\lambda^a)_{il} = \delta^{jk} = 2\left(N - \frac{1}{N}\right)\delta_{il} = \frac{2}{N}(N^2-1)\delta_{il}$

$$(\lambda^a\lambda^b\lambda^a)_{il} = (\lambda^b)_{jk} = 0 - \frac{2}{N}\lambda^b_{il}$$

$$(\lambda^a\lambda^b\lambda^a\lambda^b)_{il} = -\frac{4}{N^2}(N^2-1)\delta_{il}$$

$$(\lambda^a\lambda^a\lambda^b\lambda^b)_{il} = \frac{4}{N^2}(N^2-1)^2\delta_{il}$$

To get $C_2(\text{adj})$

$$[\lambda^a,\lambda^b] = 2i f^{abc}\lambda^c$$

$$\Rightarrow [\lambda^a,\lambda^b][\lambda^a,\lambda^b] = (2i)^2 f^{abc}f^{abd}\lambda^c\lambda^d = (2i)^2 C_2(\text{adj})\lambda^c\lambda^c$$

$$= -4\, C_2(\text{adj})\,\frac{2}{N}(N^2-1)$$

$$[\lambda^a,\lambda^b][\lambda^a,\lambda^b] = 2\left[\lambda^a\lambda^b\lambda^a\lambda^b - \lambda^a\lambda^a\lambda^b\lambda^b\right]$$

$$= 2 * \left(-\frac{4}{N^2}(N^2-1) - \frac{4}{N^2}(N^2-1)^2\right) = -\frac{8}{N}(N^2-1)\, C_2$$

$$\Rightarrow C_2 = N$$

For fund. rep

$$\left(\frac{\lambda^a}{2}\frac{\lambda^a}{2}\right)_{il} = C_2(\text{fund})\,\delta_{il} \qquad \Rightarrow \quad C_2(\text{fund}) = \frac{1}{2N}(N^2-1)$$

For later

$$(\lambda^a\lambda^b\lambda^a) = 4\left(C_2(\text{fund}) - C_2(\text{adj})\right)\lambda^b$$