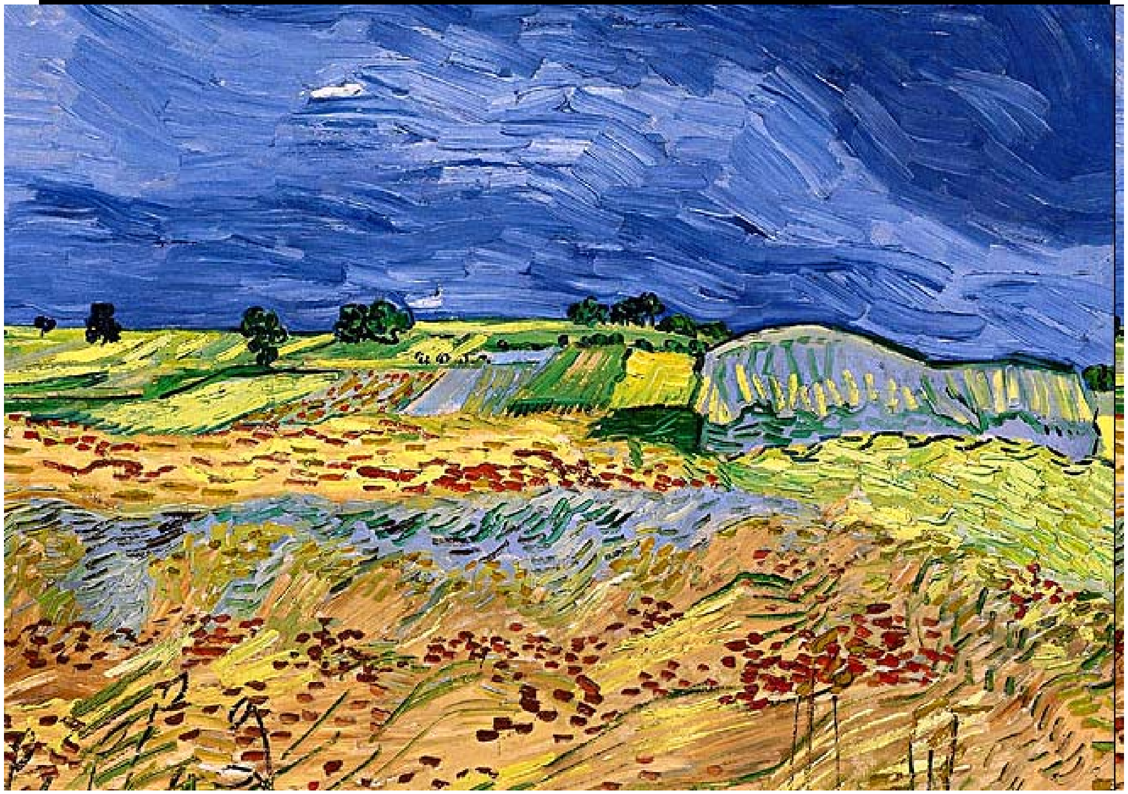
P890A- Quantum Field Theory II



Review and extension of basic ideas - 2

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9/10/2009

Pert theory - functional

$$e^{i\int d^4x\,[\mathcal{L}_0+\mathcal{L}_I(\phi,\phi^*)+J\phi+J^*\phi^*]}$$

$$= e^{i\int d^4y\,\mathcal{L}_I(\phi,\phi^*)}\, e^{i\int d^4x\,[\mathcal{L}_0+J\phi+J^*\phi^*]}$$

$$= e^{i\int d^4y\,\mathcal{L}_I\left(-i\frac{\delta}{\delta J},\,-i\frac{\delta}{\delta J^*}\right)}\, e^{i\int d^4x\,[\mathcal{L}_0+J\phi+J^*\phi^*]}$$

Then

$$\int [d\phi][d\phi^*]\, e^{i\int d^4x\,[\mathcal{L}_0+\mathcal{L}_I+J\phi+J^*\phi^*]}$$

$$= e^{i\int d^4y\,\mathcal{L}_I\left(-i\frac{\delta}{\delta J},\,-i\frac{\delta}{\delta J^*}\right)}\, N e^{i\int d^4y\,d^4z\,J^*(x)\,D_F(y-z)\,J(z)}$$

No external $\phi \Rightarrow J=0$ after diff.

$\frac{\delta}{\delta J}\frac{\delta}{\delta J^*} \Longrightarrow iD_F(\,\cdot\,)$ in pairs

$$Z[J_\mu] = \int [dA_\mu] \Big(1 + i \int d^4y \, e^2 A_\mu A^\mu D_F(y-y)$$
$$- \frac{1}{2} \int d^4y \, d^4z \, (-ie) A_\mu(y) (-ie) A_\nu(z) \left[\partial_\mu D_F(y-z) \partial^\nu D_F(z-y) + \cdots \right] \Big)$$
$$\times \, e^{i \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu \right]}$$

Use functional trick

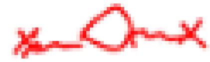
$$A_\mu(y) \, e^{i[\;]} = \frac{i\delta}{\delta J_\mu(y)} e^{i[\;]}$$

Do A integral

$$Z[J_\mu] = \left(1 + i \int d^4y \, e^2 D_F(y-y) \frac{\delta}{\delta J_\mu(y)} \frac{\delta}{\delta J^\mu(y)} + \cdots \right) Z_0[J_\mu]$$

Finally

$$G^{(2)}_{\mu\nu}(x_1, x_2) = \frac{-i \, \delta^2 Z[J_\mu]}{Z[0] \, \delta J_\mu(x_1) \, \delta J_\nu(x_2)} \Big|_{J=0}$$

=  +  + 

$iD_{F\mu\nu}(x_1 - x_2)$

Use

$$\int [d\phi]\, e^{i\int d^4x \frac{1}{2}\phi \mathcal{O} \phi} = N\left[\det \mathcal{O}\right]^{-1/2}$$

Vac. Pol

$$Z[J_\mu, J] = \int dA\, d\phi\, d\phi^*\, e^{i\int d^4x\left[\mathcal{L}_0 + \mathcal{L}_I + J_\mu A^\mu + \cancel{J\phi + J^*\phi}\right]}$$

no external ϕ $\Rightarrow$ set $J = 0$

$$= \int dA\, e^{i\int d^4x\left[-\frac{1}{4}F^2 + J_\mu A^\mu\right]} \int d\phi\, d\phi^*\, e^{i\int d^4x\, \mathcal{L}(\phi, \phi^*, A)}$$

$$\mathcal{L}(\phi, \phi^*, A) = (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi$$

$$= \underbrace{(\partial_\mu - ieA_\mu)}_{\text{int by parts}} \phi^* \quad (\partial_\mu + ieA_\mu)\phi \; - m^2 \phi^* \phi$$

$$= -\phi^* (\partial^\mu + ieA^\mu)(\partial_\mu + ieA_\mu)\phi - m^2 \phi^* \phi$$

$$= -\phi^* (D_\mu D^\mu + m^2)\phi \longrightarrow -\phi^* \mathcal{O} \phi$$

$$\int d\phi \, d\phi^* \, e^{-i\int d^4x \, \phi^* (D^2 + m^2)\phi} = N[\det(D^2 + m^2)]^{-1} \quad \leftarrow \text{not } -\tfrac{1}{2}$$

$$= e^{iW[A]}$$

$$\mathcal{L}(\phi, \phi^*, A) = (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi$$

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$$= -\phi^* (D_\mu D^\mu + m^2)\phi \rightarrow -\phi^* \mathcal{O} \phi$$

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$$= e^{iW[A]}$$

$$\left[\det(D^2+m^2)\right]^{-1} = e^{-\int d^4x \langle x|\ln(D^2+m^2)|x\rangle}$$

<u>Pert Theory</u>

$$\mathcal{O}_0 = (\Box + m^2)$$

$$D^2+m^2 = \Box + m^2 + V = \mathcal{O}_0(\mathcal{O}_0^{-1})(D^2+m^2)$$

$$= \mathcal{O}_0(1+\mathcal{O}_0^{-1}V)$$

with $\mathcal{O}^{-1} = D_F$

$$\det(D^2+m^2)^{-1} = \det \mathcal{O}_0^{-1} \det(1+\underbrace{\mathcal{O}_0^{-1}}_{D_F}V)^{-1} = N \det(1+D_F V)^{-1}$$

$$= N e^{-\int d^4x \langle x|\ln(1+D_F V)|x\rangle}$$

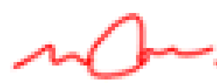
$$= N e^{-i\int d^4x\left[\langle x|D_F V|x\rangle - \frac{1}{2}\langle x|D_F V D_F V|x\rangle + \cdots\right]}$$

$$D^2 = \partial^2 + \underbrace{ie A_\mu \partial^\mu + ie \partial_\mu A^\mu - e^2 A_\mu A^\mu}_{V}$$

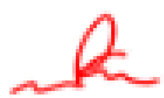
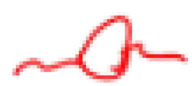
Look for $O(e^2)$

$$\langle x | D_F V | x \rangle = -e^2 A_\mu A^\mu(x)\, D_F(x-x) \qquad e$$

$$\int d^4y \langle x | D_F V | y \rangle \langle y | D_F V | x \rangle = \int d^4y\, D_F(x-y)\,(A_\mu \partial^\mu + \partial_\mu A^\mu)_y\, D_F(y-x)\,(A_\nu \partial^\nu + \partial A)_x$$

 $e^{iq\cdot x}$ $\qquad (2k+q)_\mu \qquad e^{ik\cdot x}$

$$e^{iW[A_\mu]} = e^{i\left\{\int d^4x\, D_F(x-x) A_\mu(x) A^\mu(x) + \frac{1}{2}\int d^4x\, d^4y\, (e\partial_\mu A^\mu + A\partial_\mu)_x D_F(x-y)\, e(\partial_\lambda A^\lambda + A_\lambda \partial)_y D_F(y-x)\right\}}$$

$=$  $+$ 

$$Z[J_\mu] = \int dA \, e^{i\int d^4x\left[-\frac{1}{4}F^2 + J_\mu A^\mu\right]} \quad e^{iW[A]}$$

Renormalization + Effective $\mathcal{L}$

- Large mass

$$\Pi^{\mu\nu}(q^2) = (q^\mu q^\nu - g^{\mu\nu} q^2)\,\Pi(q^2)$$

$$\Pi(q^2) = \Pi(0) + \frac{\alpha}{60\pi}\frac{q^2}{m^2}$$

In matrix elements

$$i\partial^\lambda \to q^\lambda$$

$$e^{iW[A]} = e^{i\int d^4x\left[\frac{1}{2}A_\mu\left(\partial^\mu\partial^\nu - g^{\mu\nu}\Box\right)A_\nu \Pi(0) - \frac{\alpha}{120\pi}A_\mu\left(\partial^\mu\partial^\nu - g^{\mu\nu}\Box\right)\frac{\Box}{m^2}A_\nu(x)\right]}$$

$$\frac{1}{2} A_\mu (\Box g^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Overall

$$Z[J_\mu] = \int [dA]\, e^{i\int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} (1 - \Pi(0)) - \frac{1}{120\pi} F_{\mu\nu} \frac{\Box}{m^2} F^{\mu\nu} \right]}$$

Two results

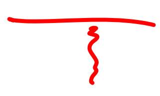
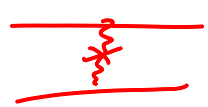
1) Renormalization $A_\mu \to A^r_\mu Z_3^{1/2}$ with $Z_3 = \frac{1}{1-\Pi(0)}$

$[dA] \to N[dA^r]$

2) Effective $\mathcal{L}$ $-\frac{1}{120\pi} F_{\mu\nu} \frac{\Box}{m^2} F^{\mu\nu}$

In pert theory

 $= -i \frac{\alpha}{60\pi} \frac{q^2}{m^2} (q^\mu q^\nu - g^{\mu\nu} q^2)$

 $+$ 

Appelquist Carrazone Thm

- Heavy fields
  1) renorm constants
  2) suppressed by $\frac{1}{M^n}$

Note if $m^2 << q^2$ cannot use eff $\mathcal{L}$

$$\Pi(q^2) = \frac{\alpha}{12\pi} \ln -q^2/m^2$$

- $\ln \square$ ill defined
- $\ln -q^2 = \ln|q^2| + i\pi\,\theta(q^2)$ — eff $\mathcal{L}$ has to be real

Physical reason
- heavy mass non-local $\Rightarrow$ local $\mathcal{L}_{eff}$
- mass light  - not local $\Rightarrow$ non-local $\mathcal{L}_{eff}$