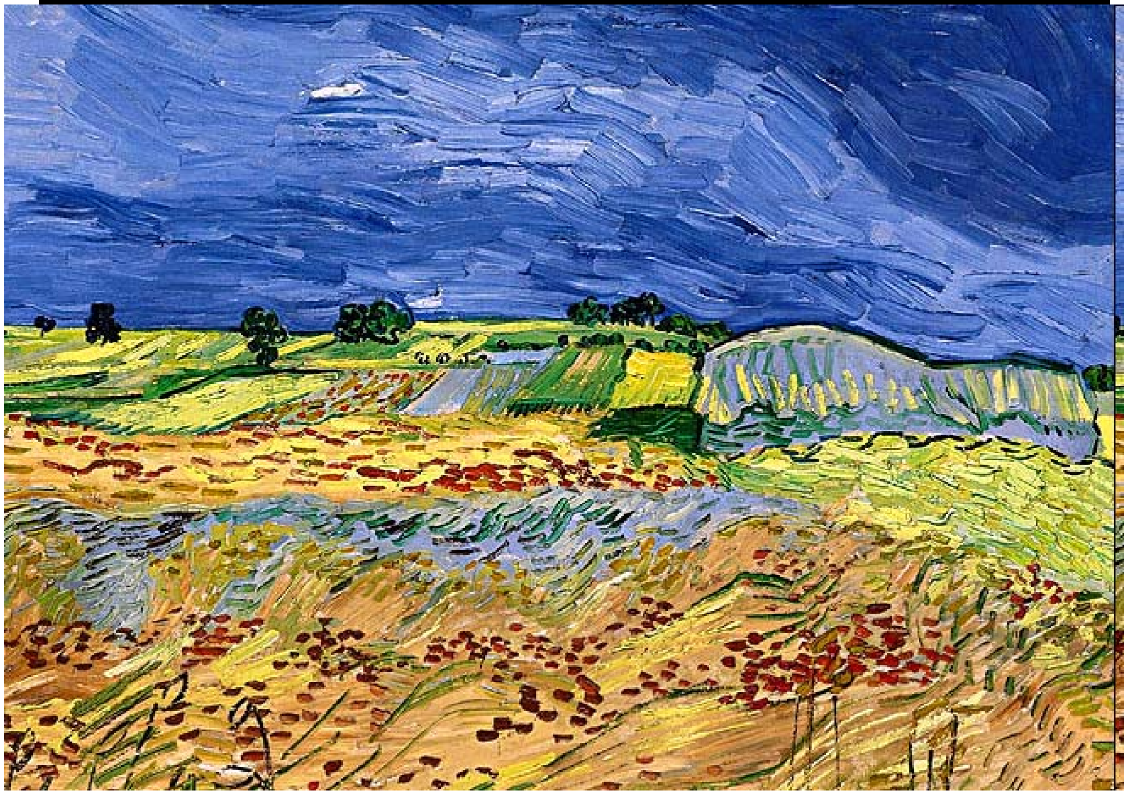
P890A- Quantum Field Theory II



Review and extension of basic ideas

9/8/2009

Review

1) Canonical

states $|p\rangle = a^{\dagger}(p)|0\rangle$

field $\phi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega} \left[a(p) e^{-ip\cdot x} + a^{\dagger}(p) e^{+ip\cdot x} \right]$

$U_I(\infty, -\infty) \Rightarrow$ Feynman rules

2) Feynman rules

X $\Rightarrow -i\mathcal{M} = -6i\lambda$

$\frac{i}{p^2 - m^2}$

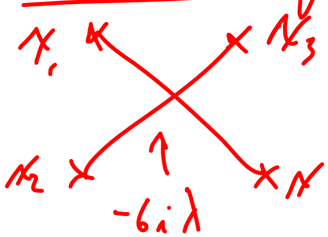
3) P.I.

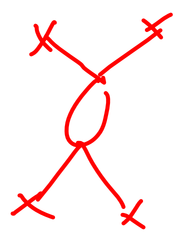
$$Z[J] = \int [d\phi]\, e^{i \int d^4x \left[\mathcal{L}(\phi, \partial_\mu \phi) + J\phi\right]}$$ vac-vac.

4) Matrix elements

$$G^{(n)}(x_1, \dots x_n) = \frac{(-i)^n}{Z[0]} \frac{\delta^n Z[J]}{\delta J(x_1) \dots \delta J(x_n)} \Big|_{J=0} = \langle 0 | T(\phi(x_1) \dots \phi(x_n)) | 0 \rangle$$

5) Pert theory





$+ \dots$

Fourier transform + drop external propagators $\Longrightarrow -i\mathcal{M}$

$$\text{F.T. } G^{(4)}(x_1 \dots x_4) = \frac{i}{p_1^2 - m^2} \frac{i}{p_2^2 - m^2} \frac{i}{\quad} \frac{i}{\quad} \, -i\mathcal{M}$$

6) Connection - LSZ reduction

$$-i\mathcal{M} = \langle 0 | a(p_3)\, a(p_4) \;\; U_I(\infty, -\infty) \qquad a^\dagger(p_1)\, a^\dagger(p_2) | 0 \rangle$$

$a \sim \phi$

$$\rightarrow \prod_i \underbrace{\int d^4x\, e^{ip_3 \cdot x_3}}_{\text{F.T.}} \underbrace{(\Box_{x_3} + m^2)}_{\text{Drop propagators}} (\cdots) \qquad G^{(4)}(x_1 \ldots x_4)$$

- rarely used

7) Techniques

- Noether's thm Symmetry J_μ, Q
- Loops
- Renormalization
  - identifying physical parameters
  - side effect - no infinities

Vacuum Polarization — 3 way

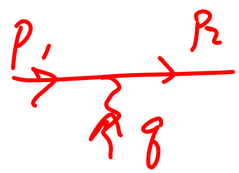
1) Feynman rules
2) P.I. $\frac{\delta Z}{\delta J}$ . . .
3) P.I. $\det[D^2 + m^2]$

Charged scalar Field $\quad \phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

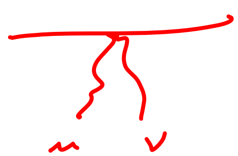
$$\mathcal{L} = (D_\mu \phi)^* (D_\mu \phi) - m^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

e

$$D_\mu = \partial_\mu + i q A_\mu$$

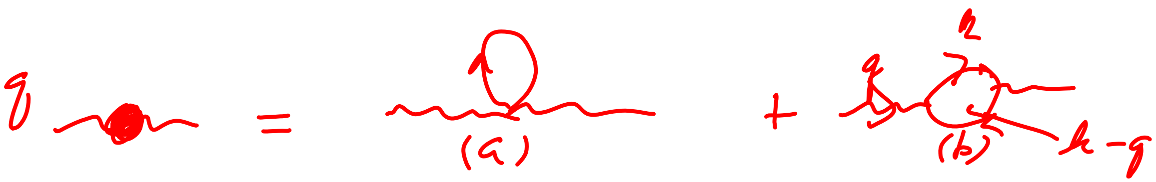
Rules 

$= -ie(p_1 + p_2)^\mu = -ie(2p_1^\mu + q^\mu)$



$= +2ie^2 g_{\mu\nu}$

Propagator



$$(a) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2} \cdot 2ie^2 g_{\mu\nu}$$

$$(b) = \int \frac{d^4k}{(2\pi)^4} -ie\,(2k+q)^\mu \frac{i}{k^2-m^2} \frac{i}{(k-q)^2-m^2} (-ie)(2k+q)^\nu$$

<u>Results</u>

$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\,\Pi(q)$$

$$\Pi(q) = \Pi(0) + \hat{\Pi}(q)$$

$$\hat{\Pi}(q) = \begin{cases} \dfrac{\alpha}{60\pi}\dfrac{q^2}{m^2} & q^2 \ll m^2 \\[2ex] \dfrac{-\alpha}{12\pi} \ln\left(-q^2/m^2\right) & q^2 \gg m^2 \end{cases}$$

Renormalization

$$\underset{\text{bare}}{A_\mu} = Z_3^{1/2} \underset{\text{ren}}{A_\mu^{(r)}}$$

$$Z_3 = 1 + \Pi(0) + \cdots = \frac{1}{1-\Pi(0)}$$

Review of PI

- finite dim integral

$$Z[J] \equiv \int dx_1 \ldots dx_N \, e^{-i\left[\frac{1}{2} x_i A_{ij} x_j + J_i x_i\right]}$$

$$= (2\pi i)^{N/2} [\det A]^{-1/2} e^{-\frac{i}{2} J_i A^{-1}_{ij} J_j}$$

$\Longrightarrow$ Field theory

$x_i \to \phi(x)$ $\qquad \sum_i \to \int d^4x$ $\qquad \int dx_1 \ldots dx_N = \int [d\phi]$

$A = (\Box + m^2)$ $\qquad A^{-1}_{ij} \to D_F(x-y)$

2 Basic results

1) $\int [d\phi] \, e^{i\int d^4x \, \phi \, \mathcal{O} \, \phi} = N [\det \mathcal{O}]^{-1/2}$

2) $Z[J] = \int [d\phi] \, e^{i\int d^4x \left[\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - m^2\phi^2 + J\phi\right]}$

$$= N \exp\left[-\frac{i}{2}\int d^4x \, d^4y \, J(x) \, D_F(x-y) \, J(y)\right]$$

Vac Pol by Functional Diff (#2)

Want

$$G^{(2)}_{\mu\nu}(x_1,x_2) = \frac{(-i)^2}{Z[0]} \frac{\delta^2 Z[J_\mu]}{\delta J_\mu(x_1)\,\delta J_\nu(x_2)}$$

Free field

$$Z[J_\mu] = \int [dA_\mu]\, e^{i\int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J_\mu A^\mu\right]}$$

Write $\int d^4x\, -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \int d^4x\, \frac{1}{2} A_\mu \mathcal{O}^{\mu\nu} A_\nu$

int by parts

$F_{\mu\nu}$: $\partial_\mu A_\nu - \partial_\nu A_\mu$

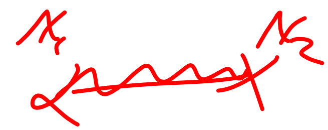
$\mathcal{O}^{\mu\nu}$: $(\Box g^{\mu\nu} - \partial^\mu \partial^\nu)$

$\mathcal{O}^{\mu\nu}$ has no inverse!
- after gauge fixing $\mathcal{O}_{\mu\nu} \to \mathcal{O}'_{\mu\nu} = \Box g_{\mu\nu}$ — inverse $D_{F\,\mu\nu}$

Then

$$Z_0[J_\mu] = N e^{-\frac{i}{2}\int d^4x\, d^4y\, J_\mu(x)\, D_F^{\mu\nu}(x-y)\, J_\nu(y)}$$

$$G^{(2)}(x_1, x_2) = i D_F(x_1 - x_2)$$



Full Theory

$$Z[J_\mu, J] = \int [dA][d\phi][d\phi^*] \, e^{i\int d^4x \left[-\frac{1}{4}F^2 + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi + J_\mu A^\mu + J\phi + J^* \phi^*\right]}$$

$$= \int [dA] \, e^{i\int d^4x \left[-\frac{1}{4}F^2 + J_\mu A^\mu\right]} \underbrace{\int d\phi \, d\phi^* \, e^{i\int d^4y \left[(D\phi)^2 - m^2\phi^2 + J\phi + J^*\phi^*\right]}}_{e^{iS_\phi}}$$

<u>Now</u>

$$(D_\mu \phi)^* D^\mu \phi = -ie A_\mu \underbrace{\left(\phi^* \partial_\mu \phi - (\partial_\mu \phi^*)\phi\right)}_{J^\phi_\mu} + e^2 A_\mu A^\mu \phi^* \phi + \partial_\mu \phi^* \partial^\mu \phi$$

<u>Expand + keep $\mathcal{O}(e^2)$</u>

$$e^{iS_\phi} = \left(1 + i\int d^4y \, e^2 A_\mu A^\mu \phi^* \phi + \frac{1}{2}\int d^4y \, d^4z \, (-ie A_\mu(y) J^\mu_\phi(y)) (-ie A_\nu(z) J^\nu_\phi(z)) + \ldots\right)$$

$$\times \, e^{i\int d^4x \left[\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi + J\phi + \phi^* J^*\right]}$$

Functional trick

$$\phi(y)\, e^{iS[\quad]} = -i\frac{\delta}{\delta J(y)}\, e^{iS[\quad]}$$

So that

$$\int[d\phi][d\phi^*]\left(1+\int d^4y\cdots\phi\cdots-\frac{1}{2}\cdots(\quad)\right)e^{i\int d^4x[\quad]}$$

$$=\left(1+i\int d^4y\, A_\mu A^\mu(y)\frac{\delta^2}{\delta J(y)\,\delta J^*(y)}-\frac{1}{2}\int d^4y\, d^4z\,(-ie)^2 A_\mu(y)A_\nu(z)J_\mu\!\left(\frac{\delta}{\delta J}\right)J_\nu\!\left(\frac{\delta}{\delta J}\right)\right)$$

$$\times\underbrace{\int[d\phi][d\phi^*]\, e^{i\int d^4x\left[\partial_\mu\phi^*\partial^\mu\phi-m^2\phi^*\phi+J\phi+J^*\phi\right]}}_{N\exp\left[i\int d^4x\, d^4y\, J^*(x)D_F(x-y)J(y)\right]}$$

no factor of $\frac{1}{2}$

$$Z[J_\mu] = \int [dA_\mu] \Big(1 + i\int d^4y\, A_\mu A^\mu D_F(y-y)$$

$$- \frac{1}{2}\int d^4y\, d^4z\, -iA_\mu(y)\left[\partial^\mu_y D_F(y-z) D_F(z-y) + 3\,\text{term}\right] -iA_\mu(z) \Big)$$

$$\times\, e^{i\int d^4x\left[-\frac{1}{4}F^2 + J_\mu A^\mu\right]}$$