EFT 5

Note Title 10/29/2009

O.P.E.

In SM $\quad \frac{g}{2\sqrt{2}} W_\mu \cdot \left[V_{ud}\, \bar{u}\gamma^\mu(1+\gamma_5)d + V_{us}\, \bar{u}\gamma^\mu(1+\gamma_5)s + \right]$

Form

$$\mathcal{H}_W = \underbrace{\frac{g^2}{8M_W^2}}_{G_F/\sqrt{2}} \overset{V_{ud}V_{us}^*}{-M_W^2} \int d^4x\, D_F(x, M_W)\, T\left[J_\mu^{ud}(x)\, J^{*\mu\, us}(0) \right]$$

$$= \frac{G_F}{\sqrt{2}} VV\, J_\mu^{ud}(0)\, J^{us\,\mu}(0)$$

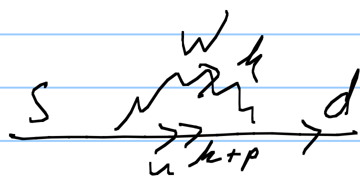
$$= \frac{G_F}{\sqrt{2}} VV \sum_i C_i(M_W/\Lambda)\, O_i(0)$$

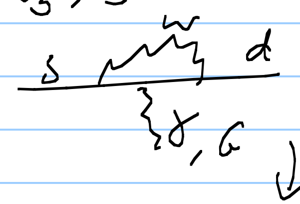
$$T(J(x)\,J(0)) = \sum_i C_i(x)\,\mathcal{O}_i(0)$$

To lowest order $\mathcal{O}_1 = J_\mu(0)\,J^\mu(0)$

$$C_1 = 1$$

<u>Aside</u> Dimension 4 operator $\mathcal{O}_* = \bar{d}\,\not{D}(1+\gamma_5)s$

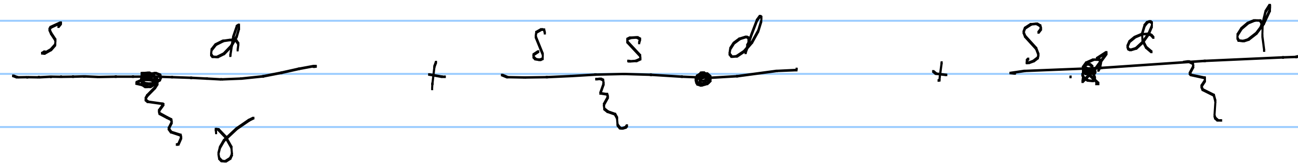




$$(\ )\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-M_W^2}\,\gamma_\mu(1+\gamma_5)\,\frac{\not{k}+\not{p}}{(k+p)^2}\,\gamma^\mu(1+\gamma_5) = \#\,\not{p}(1+\gamma_5)$$

$$\Rightarrow\ \bar{d}\,\not{\partial}(1+\gamma_5)s \longrightarrow \bar{d}\,\not{D}(1+\gamma_5)s$$

Why not included? $s \to d G$, $s \to d\gamma$



$= 0$

Also for $\bar{d} s$

~~Cont~~ Does not contribute to anything   Feinberg Kabir Weinberg

Can remove it by diagonalization

$$\mathcal{L} = \bar{s}\, i\not{D}\, s + \bar{d}\, i\not{D}\, d + a\, \bar{d}\, i\not{D}\, s + a^{*}\, \bar{s}\, i\not{D}\, d$$

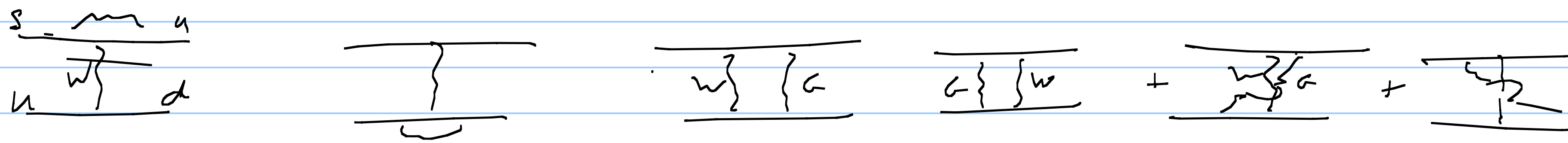
$$= (\bar{s}\ \bar{d}) \begin{pmatrix} 1 & a \\ a^{*} & 1 \end{pmatrix} \begin{pmatrix} s \\ d \end{pmatrix}$$

$\Rightarrow$ rotate fields $\Big(\quad \Big) \to \begin{pmatrix} 1+a & \\ & 1-a \end{pmatrix}$

$\Rightarrow$ rescale $s \to \frac{1}{\sqrt{1+a}}\, s$

$\Rightarrow$ all effects vanish

OPE to $O(\alpha_s)$



(a) (b) c

a & b don't contribute $\left(Z_2^{-1/2}\right)^2 (Z_1)\ \gamma^\mu_{(1+\gamma_5)} \to \gamma^\mu (1+\gamma_5)$

$$\mathrm{Amp}_c = (\ ) \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - M_W^2} \frac{-i}{k^2} \left[-ig\gamma_\lambda \frac{\lambda^a}{2} \frac{\not k}{k^2} \gamma_\mu (1+\gamma_5) \right] \left[-ig\gamma^\lambda \frac{\lambda^a}{2} \frac{\not k}{k^2} \gamma^\mu (1+\gamma_5) \right]$$

$$I = \int_\Lambda^\infty \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \frac{1}{k^2 - M_W^2} = \frac{-i}{16\pi^2} \frac{1}{M_W^2} \ln M_W^2/\Lambda^2$$

separate high energy from low

$$\gamma_\lambda \gamma_\sigma \gamma_\mu = g_{\lambda\sigma}\gamma_\mu + g_{\sigma\mu}\gamma_\lambda - g_{\mu\lambda}\gamma_\sigma - i\,\varepsilon_{\lambda\sigma\mu\nu}\gamma^\nu\gamma_5$$

$$\Rightarrow O_2 = \bar{d}\,\gamma^\mu(1+\gamma_5)\lambda^a u\;\bar{u}\,\gamma_\mu(1+\gamma_5)\lambda^a d$$

Then

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} VV\Big[\underbrace{O_1(0)}_{\uparrow\, C_1=1} - \underbrace{\frac{3g^2}{32\pi^2}\ln M_W^2/\Lambda^2}_{\uparrow\, C_2(M_W/\Lambda)}\, O_2(0)\Big] + \cdots$$

Integrated out high energy gluons $E > \Lambda$

$\Rightarrow \mathcal{L}_{eff}(\Lambda)$

$\Rightarrow$ when using include gluons up to $E = \Lambda$

- lattice $\frac{1}{a} \sim \Lambda$

Wilsonian EFT

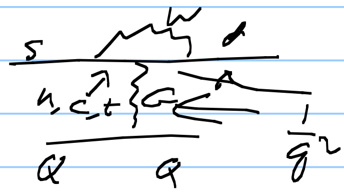
Rest of story

1) At order α_s

$\Rightarrow \bar{d}\, \not{D}(1+\gamma_5)\, s \Rightarrow$ drop

$\rightarrow$

$= \bar{d}\, \sigma^{\mu\nu}(1-\gamma_5)\lambda^A s\, F^A_{\mu\nu}\, m_s$

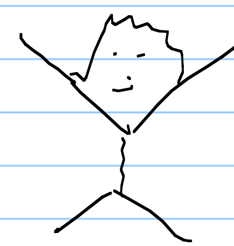


$\gamma_\mu(1+\gamma_5)\, q^2$

$\frac{1}{q^2}$

$\bar{d}\, \gamma^\mu (1+\gamma_5) \lambda^a s\ \bar{Q}\, \gamma_\mu \lambda^a Q$ "penguin"

~~CP~~



2) Technology R.G.

$$C_i(M,\Lambda) \rightarrow \left(\frac{\alpha_s(\Lambda)}{\alpha_s(M_W)} \right)^{d_i}$$

Background Field Method

Logic $(\lambda\phi^4)$

- identify "quantum fields" $\Rightarrow$ into loop (rest are "background field")
- do loop
- then let fields act on external states

$$\phi = \bar{\phi} + \phi_q$$

do PI over $\phi_q \implies W[\bar{\phi}]$

at n external states

Procedure:

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - \frac{\lambda}{4}\phi^4$$

let $\phi = \bar{\Phi} + \phi_q$

$$\mathcal{L} = \mathcal{L}(\bar{\Phi}) - \overbrace{\left[(\Box + m^2)\bar{\Phi} + \lambda\bar{\Phi}^3\right]}^{= 0 \ \text{Eq of motion}} \phi_q$$

$$+ \left(\frac{1}{2}\left((\partial_\mu \phi_q)^2 - m^2\phi_q^2\right) - 3\lambda\bar{\Phi}^2\phi_q^2\right) + \cdots$$

$$= \mathcal{L}(\bar{\Phi}) - \frac{1}{2}\phi_q \, \mathcal{D} \, \phi_q$$

$$\mathcal{D} = \Box + m^2 + \underbrace{6\lambda\bar{\Phi}^2}_{\sigma}$$

D_0 P.I

$$\int [d\phi] = \int [d\phi_q]$$

$$Z = N [\det \mathcal{D}]^{-1/2} e^{i\int d^4x \, \mathcal{L}(\bar{\phi})}$$

$$= N e^{i\int d^4x \left[\mathcal{L}(\bar{\phi}) - \frac{1}{2}\langle x | \ln \mathcal{D} | x\rangle\right]}$$

1) Heat kernal

$$= N e^{i\int d^4x \left[\mathcal{L}(\bar{\phi}) + \frac{1}{2}\frac{1}{(4\pi)^{d/2}} \sum_n m^{d-2n} \Gamma(n - d/2)\, a_n\right]}$$

$a_0 = 1$, $a_1 = -\sigma$, $a_2 = -\frac{1}{2}\sigma^2$

$$\int d^4x \left[\mathcal{L}(\bar\Phi) - \underbrace{\frac{m^2}{32\pi^2}\,\Gamma(1-d/2)\,3\lambda\,\bar\phi^2}_{\text{mass renorm}} - \underbrace{\frac{1}{32\pi^2}\,\Gamma(2-d/2)\,\bar\phi^4\,(6\lambda)^2}_{\lambda\ \text{r.}} + \cdots \right]$$

$\Rightarrow$ renorm easily

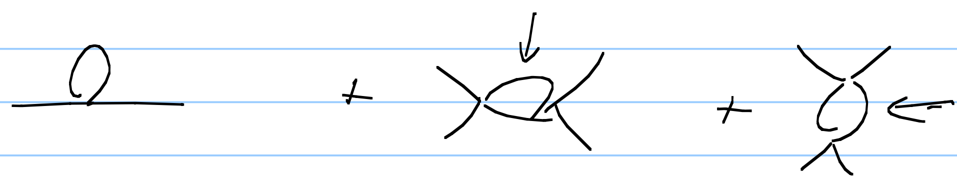
- explicit Φ fields

2) Pert theory $\quad \mathcal{D} = \mathcal{D}_0 + \sigma \qquad \mathcal{D}_0 = \Box + m^2$

$$\ln \mathcal{D} = \ln \mathcal{D}_0 (1 + \mathcal{D}_0^{-1}\sigma)$$

$$= \ln \mathcal{D}_0 + \underbrace{\ln (1 + \mathcal{D}_0^{-1}\sigma)}_{\mathcal{D}_0^{-1}\sigma + \frac{1}{2}\mathcal{D}_0^{-1}\sigma \mathcal{D}_0^{-1}\sigma \cdots}$$

$$\mathcal{L} = \int d^4x \left[-D_F(x-x)\,\sigma(x) + \frac{1}{2}\int d^4y\; D_F(x-y)\,\sigma(y)\,D_F(y-x)\,\sigma(x) + \cdots \right]$$



Take matrix elements $\Rightarrow$ pert theory

<u>Difference</u> –

– Heat kernel – good for renorm, bad for finite parts

– Pert $\rightarrow$ renorm process by process, reproduce pert theory

B.F. renorm of σ model

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)$$

$$U = e^{i\vec{\tau}\cdot\vec{\pi}/v}$$

$\vec{\tau}\cdot\Delta^a$

Various way $\qquad U = \bar{U} e^{i\Delta} \qquad U^\dagger U = 1$

$$= e^{i\Delta'}\bar{U} \quad = \xi e^{i\eta} \xi \qquad \xi\xi = U$$

Expand

$$\mathcal{L} = \mathcal{L}(\bar{U}) - 2i\, \mathrm{Tr}\left(\bar{U}^\dagger \partial_\mu U\, \partial_\mu \Delta\right)$$

Int. by parts $\Rightarrow \partial^\mu \bar{U}^\dagger \partial_\mu U = 0$

Eq of Motion

$$+ \mathrm{Tr}\left[\partial_\mu \Delta \partial^\mu \Delta + \bar{U}^{\dagger} \partial_\mu \bar{U}\left(\Delta \partial_\mu \Delta - (\partial_\mu \Delta)\Delta\right)\right]$$

Write in form

$$\mathrm{Tr}[\ ] = 2\,\Delta^a\,\mathcal{D}^{ab}\,\Delta^b$$

$$\mathcal{D}^{ab} = (d_\mu d^\mu)^{ab} + \sigma^{ab} \qquad d_\mu^{ab} = \delta^{ab}\partial_\mu + \Gamma_\mu^{ab}$$

$$\Gamma_\mu^{ab} = -\frac{1}{4}\,\mathrm{Tr}\left([\tilde{\tau}^a, \tilde{\tau}^b]\,\bar{u}^\dagger \partial_\mu \bar{u}\right)$$

$$\sigma^{ab} = \frac{1}{8}\,\mathrm{Tr}\left([\tilde{\tau}^a, \bar{u}^\dagger \partial_\mu \bar{u}][\tilde{\tau}^b, \bar{u}^\dagger \partial_\mu \bar{u}]\right)$$

P.I.

$$\int [d\Delta]\, e^{i\int d^4x \frac{F^2}{2}\Delta^a (\mathcal{D})^{ab}\Delta^b} = e^{iW}$$

$\Rightarrow$ Heat kernal $\swarrow$ Tr over x $\qquad$ $\swarrow$ Trace over a,b

$$-iW = \frac{-i}{2(4\pi)^{d/2}} \int d^4x \quad \Gamma(2 - d/2) \; \mathrm{Tr}\left(\frac{1}{12} \underbrace{\Gamma_{\mu\nu}\Gamma^{\mu\nu}} + \frac{1}{2}\sigma^2 \right)$$

$$W = \int d^4x \; \frac{-1}{192\pi^2}\left(\frac{2}{d-4} - \ln 4\pi + \gamma\right)\left[\frac{1}{2}\left[\mathrm{Tr}\left(\partial_\mu \overline{U}\, \partial^\mu \overline{U}^\dagger\right)\right]^2 + \mathrm{Tr}\left(\partial_\mu \overline{U}\, \partial_\nu \overline{U}^\dagger\right) \mathrm{Tr}\left(\partial_\mu \overline{U}\, \partial_\nu \overline{U}^\dagger\right)\right]$$

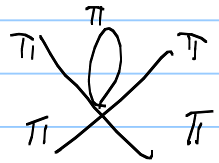
- renormalize α_1, α_2 $\qquad$ $\uparrow$ explicit I

- all reactions have same renorm.!

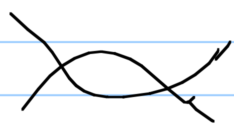
Use B.F in "our" HW calc?

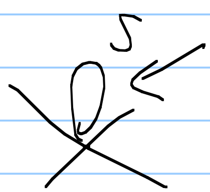
$$\int d^4k \, \frac{1}{k^2-m^2} \sim m^2 \to 0$$

$= 0$ in dim reg

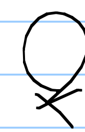


→ expand to $O(\phi^6)$
contract all combinations





$S^2 \underbrace{Tr(\partial_\mu U \partial^\mu U^+)}_{\propto}$



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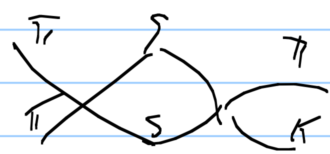

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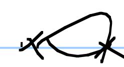

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