EFT 3

Note Title 10/22/2009

$$Z[\vec{J}_\pi] = \int dS\, d\pi\; e^{i\int d^4x\,[\underbrace{\mathcal{L}}_{\text{full}}(S,\pi) + \vec{J}\cdot\vec{\pi}]}$$

$$= \int d\pi\; e^{i\int d^4x\,[\mathcal{L}_{eff}(\pi) + \vec{J}\cdot\vec{\pi}]}$$

$\mathcal{L}_{eff}$: parameters chosen right

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu S)^2 - m^2 S^2\right] + \frac{(v^2+2vS+S^2)}{4}\,\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)\; \underbrace{-\lambda v S^3 - \frac{\lambda}{4}S^4}_{\text{neglect for moment}}$$

$$e^{iW[\pi]} = \int dS\; e^{i\int d^4x\,\mathcal{L}(S,\pi)}$$

$$\sigma(x) = \frac{1}{2}\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)$$

$$\mathcal{L} = \frac{N^2}{2}\sigma + \frac{1}{2}\left[(\partial_\mu S)^2 - (m^2+\sigma) S^2\right] - S N \sigma$$

Define

$$(\Box + m^2 + \sigma)\, D_\sigma(x-y) = -\delta^4(x-y)$$

$$\tilde{S}(x) = S(x) + \frac{1}{2}\int d^4y\, D_\sigma(x-y)\, N\sigma$$

End with

$$\int d^4x\, \mathcal{L}(S,\pi) = \int d^4x \left[\frac{N^2}{2}\sigma - \frac{1}{2}\tilde{S}\left[\Box + m^2 + \sigma\right]\tilde{S}\right] - \frac{1}{4}\int d^4x\, d^4y\, N\sigma\, D(x-y)\, N\sigma(y)$$

Heat kernal

$$\int d\tilde{S}\, e^{i\int d^4x\, \tilde{S}\, \mathcal{D}\, \tilde{S}} = \left[\det \mathcal{D}\right]^{-1} = e^{\int d^4x \langle x| \ln \mathcal{D} |x\rangle}$$

$$= e^{\int d^4x \frac{i}{(4\pi)^{d/2}} \sum_n \underbrace{m^{d-2n}\,\Gamma(n-d/2)}_{\frac{\partial^n}{m^n}} a_n}$$

$$\mathcal{D} = \Box + m^2 + \sigma$$

$$a_0 = 1, \quad a_1 = -\sigma, \quad a_2 = \frac{1}{2}\sigma^2 + \frac{1}{6}\underbrace{[\partial_\mu,[\partial^\mu,\sigma]]}_{\Box\sigma}$$

$\Rightarrow$ total deriv $\Rightarrow$ drop

Also

$$D_\alpha(x-y) = \frac{1}{m^2}\delta^4(x-y) + \mathcal{O}(\sigma) + \mathcal{O}\left(\Box\,\delta^4(x-y)\right)$$

Result

$$\frac{1}{2}\ln 4\pi - \gamma - \frac{1}{2}\ln m^2/\mu^2$$

$$W[\pi] = \int d^4x \Bigg\{ \left(\frac{v^2}{2} - \frac{m^2}{32\pi^2}\left(\frac{2}{d-4} + \cdots \right) \right) \sigma$$

$$\left(\frac{v^2}{8m^2} + \frac{1}{64\pi^2}\left(\frac{2}{d-4} + \cdots \right) \right) \sigma^2 + O(\sigma \Box \sigma) + O(\sigma^3) \Bigg\}$$

1) v^2 term $\Rightarrow$ renorm of v

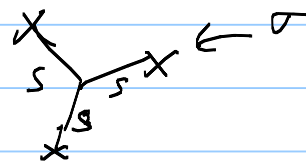
2) σ^2 term is finite in full theory

- this is answer

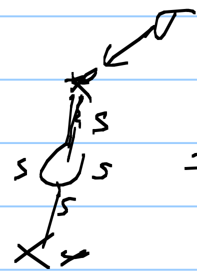
- still have to do π loops

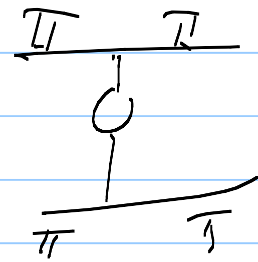
$\Longrightarrow$ end is finite

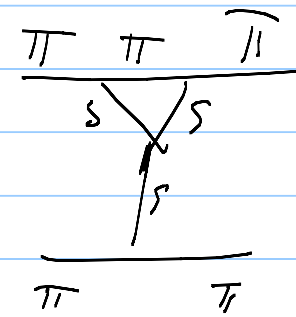
What about $\lambda v S^2 + \frac{\lambda}{4} S^4$?

Tree level 

$= -\frac{1}{m^6} 2\lambda v \times (v\sigma)^3 = -\frac{2\lambda v^4}{m^6}\sigma^3$

One loop 

$=$ 

$+$ 

Preparing for full matching

1) What is form of answer?

$$U = e^{i\vec{\tau}\cdot\vec{\pi}/v} \qquad\qquad U^\dagger U = 1$$

$$U \to L U R^\dagger$$

$$\mathrm{Tr}(U U^\dagger) = 2$$

$$\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)^2 \checkmark$$

$$\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger) \qquad , \qquad \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger)$$

$$\mathrm{Tr}(\partial_\mu U \partial^\nu U^\dagger)\, \mathrm{Tr}(\partial_\mu U \partial^\nu U^\dagger)'$$

Note only 2 comb at linear order $\qquad L_\mu = \tau^i l^i_\mu = U \partial_\mu U^\dagger$

$$a\, l^i_\mu l^{i\mu} l^{j\nu} l^j_\nu + b\, l^i_\mu l^i_\nu l^{j\mu} l^{j\nu}$$

Then

$$\mathcal{L}_{eff} = \frac{F^2}{4} \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^\dagger) + \alpha_1 \left(\mathrm{Tr}(\partial_\mu U \, \partial^\mu U^\dagger)\right)^2 + \alpha_2 \mathrm{Tr}(\partial_\mu U \, \partial_\nu U^\dagger) \mathrm{Tr}(\partial^\mu U \, \partial^\nu U^\dagger)$$

Tree level

$F = v$

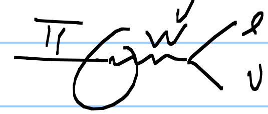
$\alpha_1 = \frac{v^2}{8m^2}$, $\alpha_2 = 0$

2) <u>What parameters</u>?

- λ, μ, v, m — 2 free parameters

- I will use v, m — λS^4
  - v in $\mathcal{L}_{eff}$
  - m physical meaning

<u>"Best" definition</u> of v — $\Sigma = (\overset{v}{v} + S)\, U$

~~in~~ SM 

$$\langle \pi^i | L^{a}_{\mu} | 0 \rangle = i v\, p^\mu\, \delta^{ij} = i F_\pi\, p^\mu\, \delta^{ij}$$

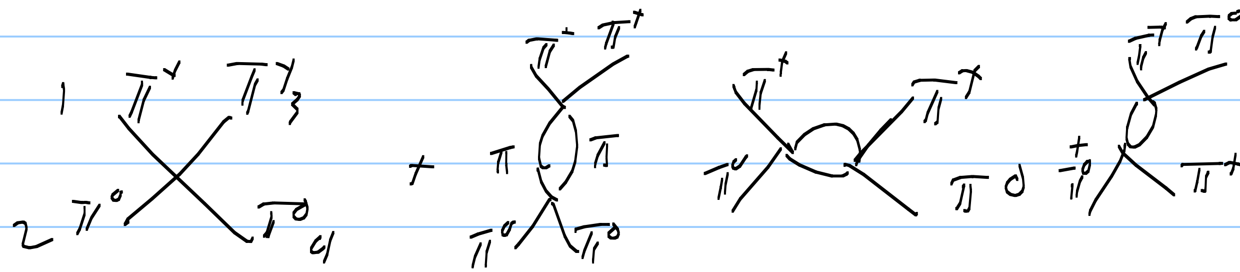
Use Noether's theorem

$$L^i_\mu = \frac{v^2}{2} \mathrm{Tr}\left(\tau^i\, U \partial_\mu U^+\right) \sim v\, \partial_\mu \pi^i + \ldots$$

Effective theory

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) = \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a + \mathcal{O}(\pi^4) + \mathcal{O}(\pi^6) + \dots$$

$\pi^+ \pi^0 \to \pi^+ \pi^0$



$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

$$\ln 4\pi - \gamma$$

$$\mathcal{M} = \frac{t}{F^2} + \frac{4}{F^4}\left[2\left[\alpha_1 + \frac{1}{96\pi^2}\left[\frac{2}{d-4} - \right] \right] t^2 \right.$$

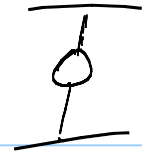
$$\left. + \left(\alpha_2 + \frac{1}{48\pi^2}\left[\frac{2}{d-4} --- \right] \right) \left(s^2 + u^2 \right) \right]$$

$$+ B(s,t,u)$$

$$B(s,t,u) = \frac{1}{6F^2}\left[3t^2 J(t) + s(s-u) J(s) + u(u-s) J(u) \right.$$

$$\left. - \frac{1}{96\pi^2}\left[21 t^2 + 5(s-u)^2 \right] \right]$$

$$J(s) = \frac{1}{16\pi^2}\left[\ln\left(-s/\mu^2 \right) + 2 \right]$$

Full theory  $+ \cdots$

$$\mathcal{M} = \frac{t}{v^2} + \frac{4}{v^4}\left\{ 2\left(\frac{v^2}{8M^2} - \frac{35}{144\pi^2}\right) t^2 - \frac{11}{72\pi^2}\left(s^2 + u^2\right) \right\} + O(s^{4}$$

$$+ B_\sigma(s,t,u)$$

B_σ is same $\quad J_\sigma = \frac{1}{16\pi}\left[\ln\frac{-s}{m^2} + 2\right]$

Define

$$\alpha_1^{ren} = \alpha_1^{bar} + \frac{1}{96\pi^2}\left[\frac{2}{d-4} + \frac{1}{2}\ln 4\pi - \gamma\right]$$

$$\alpha_2^{ren} = \alpha_2^{bar} + \frac{1}{48\pi^2}\left[\qquad\right]$$

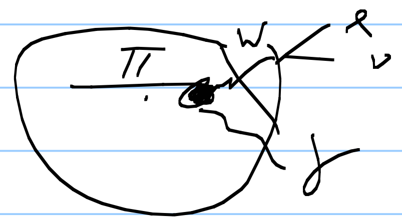
Matching

$$\alpha_1^{ren}(\mu) = \frac{v^2}{8 m_\sigma^2} + \frac{1}{24\pi^2}\left(\ln m_\sigma^2/\mu^2 - \frac{35}{6}\right)$$

$$\alpha_2^{ren}(\mu) = \frac{1}{12\pi^2}\left(\ln m_\sigma^2/\mu^2 - 11/6\right)$$

Reduced all effects of S, $\underline{v^2, \alpha_1, \alpha_2}$

- other processes, use eff theory



$\pi\pi \to 4\pi$