Effective Field Theory 1

Note Title 10/15/2009

HW

$$\mathcal{L} = KE + g\bar{\Psi}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)\Psi$$

$$\Rightarrow \beta(g) \quad \nsim \quad g^2(g^2)$$

Effective Field Theory

DGH
Weinberg
Georgi

- QFT with relevant D.O.F.
- separation of high E vs low E
- insight - relevant DOF are propagating particles

1930-50's rise QFT
60' decline of QFT
70' triumph of Renormalizeable QFT ← Wilson + Weinberg

80's - growth of EFT
90's on standard tool

Path
1) Effective $\mathcal{L}$ - QED + σ model
2) Matching at tree level
3) Matching one loop ← *
4) Explicit - ChPTh
5) advanced techniques - Wilson, QPE, Background field method

Organizing principle #1 — locality

$\Delta x \, \Delta p \sim 1 \quad \Longrightarrow$ range of interaction $\sim \dfrac{1}{\Delta E}$

Propagator

$$D_F(x-y) = -\frac{1}{m^2}\,\delta^4(x-y) + \frac{\Box\,\delta^4(x-y)}{m^4}$$

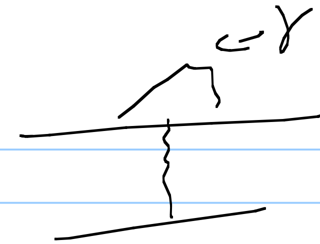
$$\lim_{m \to \text{large}} \frac{e^{-mr}}{4\pi r} = \frac{1}{m^2}\,\delta^3(x)$$

Not just exchange of heavy particles

- Also HE part of loops

 or  $= \Delta \mathcal{L} = \delta m\, \bar\psi \psi$ or $(Z-1) F_{\mu\nu} F^{\mu\nu}$ local

HE parts of loops



$$I(p,q) = \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \; \frac{1}{k^2} \; \frac{1}{[\quad]} \; \frac{1}{[\quad]}$$

↑ cutoff sensitive to HE.

$$= \underbrace{\ln\Lambda + \text{const}}_{\text{renorm.} \checkmark} + f(p,q) + \underbrace{\frac{q^2}{\Lambda^2} + \frac{p^2}{\Lambda^2}}_{\text{drop normally} \checkmark} + \cdots$$

<u>Appelquist Carrazone Thm</u> — "decoupling theorem"

- heavy scales or heavy particles

  ⟹ renormalized parameters

  or suppressed by powers of heavy scale

⟹ heavy stuff decouples

Caveats

ex. $m_t \to \infty$, breaks symmetry of SM $\begin{pmatrix} t \\ b \end{pmatrix}_L \sim SU(2)$

$\Rightarrow$ divergence that cannot be absorbed

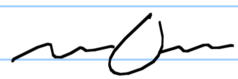
$\Rightarrow$ non decoupling

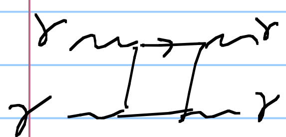
$$\frac{M_W^2 - M_Z^2}{M_Z^2} \propto m_t^2 \quad \text{not} \quad \frac{1}{m_t^2}$$

ex $m_H \to \infty$ Higgsless $SU(2)_L$ sich . . $\ln m_H^2$

but $m_t, m_b \to \infty$ decoupling works ✓

QED eff $\mathcal{L}$ — fermion heavy

 $\underbrace{\left\{\frac{1}{d-4} + \cdots\right\}}_{\text{renorm.}} + \underbrace{\frac{q^2}{m^2}}_{\text{suppressed}} + \cdots$

 finite → no renorm $\sim \frac{1}{M^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2$

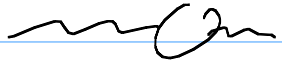
$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{60\pi M^2}F_{\mu\nu}\Box F^{\mu\nu}$$

$\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$

$$+\frac{\alpha^2}{90m^4}\left[\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{7}{16}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right]$$ ← Euler Heisenberg

- local $\mathcal{L}$, $\frac{1}{m^n}$

$$\Pi_{\mu\nu} = (q_\mu q_\nu - g_{\mu\nu} q^2)\left[\frac{\alpha}{3\pi}\left\{\frac{2}{4-d} + \cdots\right\} + \frac{\alpha}{15\pi}\frac{q^2}{m^2}\right]$$

 $\Longrightarrow$ 

$$\mathcal{L}_{eff} = \frac{\alpha}{12\pi}\{\ \} F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{60\pi m^2} F_{\mu\nu}\Box F^{\mu\nu}$$ matching

If particles are not heavy $\Rightarrow$ not local

$$\Pi(q^2) \sim \frac{\alpha}{3\pi}\ln\left(-q^2/m^2, \mu^2\right)$$

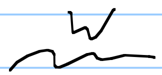
Can't match $F_{\mu\nu}\ln(\Box)F^{\mu\nu}$

$\uparrow$ $D = \pm$

$\Rightarrow$ include light as dynamical

Second Organizing Principle — Energy expansion

 $\sim \dfrac{g^2}{M^2} + \dfrac{g^4}{M^4} + \cdots$



$\sim \dfrac{1}{M_W^2}\left[1 + \dfrac{k^2}{M_W^2} + \cdots\right]$

$\uparrow\ 1\,\mathrm{GeV}^2/M_W^2 \sim 10^{-4}$

treat higher orders as perturbation

q^2/M_H^2 is small parameter

Sigma model

$$\mathcal{L} = \bar{\Psi} i \not{\partial} \Psi + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi} - V(\sigma,\pi) - g\bar{\Psi}(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma_5)\Psi$$

Goal:

$$\mathcal{L}_{eff} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \qquad U = e^{i\frac{\vec{\tau}\cdot\vec{\pi}}{v}} \qquad U \to L U R^\dagger$$

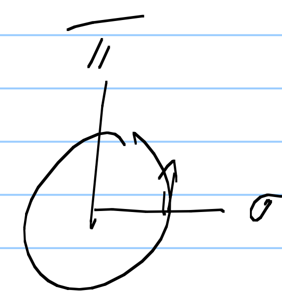
We have SSB

$$v = \sqrt{\mu^2/\lambda} \qquad V(\sigma,\vec{\pi}) = -\frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2$$

$m_\sigma = \sqrt{2}\,\mu$

$m_\Psi = g v \qquad \sigma = v + \tilde{\sigma}$

$m_\pi = 0$



$$V(\tilde{\sigma},\pi) = \lambda v \tilde{\sigma}(\tilde{\sigma}^2 + \pi^2) + \frac{\lambda}{4}(\tilde{\sigma}^2 + \pi^2)^2$$

Also $\quad \Sigma = \sigma + i\vec{\tau}\cdot\vec{\pi}$

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \frac{1}{4} \mathrm{Tr}\left(\partial_\mu \Sigma \, \partial^\mu \Sigma^+\right) - g\left(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^+ \psi_L\right)$$

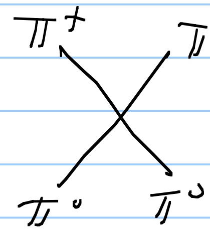
$$- \frac{\mu^2}{4} \mathrm{Tr}\left(\Sigma^+ \Sigma\right) - \frac{\lambda}{16}\left[\mathrm{Tr}\left(\Sigma^+ \Sigma\right)\right]^2$$

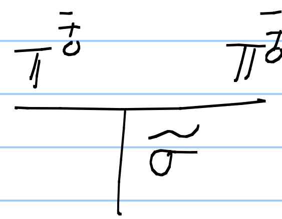
$SU(2)_L \otimes SU(2)_R$ symmetry

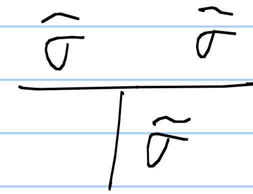
$$\Sigma \to L \Sigma R^+$$

Low energy processes $\quad \pi^+ \pi^0$

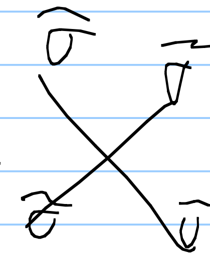
$$\mathcal{L}_I = -\lambda v \tilde{\sigma}(\tilde{\sigma}^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\tilde{\sigma}^2 + \vec{\pi}^2)^2$$





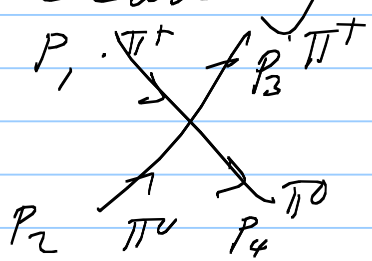


$+ \; - -$

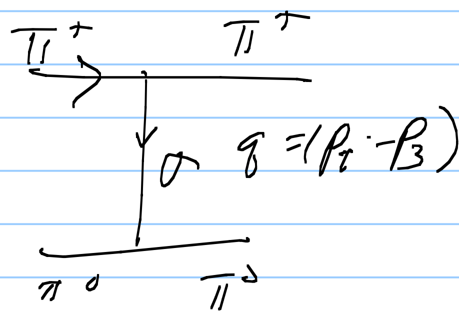


$-2i\lambda \qquad -2i\lambda v \qquad -6i\lambda v \qquad = -6i\lambda$

Scattering



$+$



$$\mathcal{M} = -2i\lambda \quad + (-2i\lambda v)^2 \frac{i}{q^2 - m_\sigma^2}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad 2\mu^2 = 2\lambda v^2$$

$$= -2i\lambda \left[1 + \frac{2\lambda v^2}{q^2 - 2\lambda v^2} \right] = \frac{-2i\lambda}{[q^2 - 2\lambda v^2]} \left[q^2 \right]$$

$$= i\frac{q^2}{v^2} \left[1 + \frac{q^2}{2\lambda v^2} + \cdots \right]$$

$$\uparrow \qquad\qquad \uparrow\ q^2/m_\sigma^2$$

Write eff $\mathcal{L}$

$$\mathcal{L}_{eff} = -\partial_\mu \pi^0 \partial^\mu \pi^0 \quad \pi^+ \pi^- \quad \frac{1}{v^2} \quad \checkmark$$

$$\langle i\mathcal{L}_{eff} \rangle = -2\,(ip_4)_\mu \cdot (-ip_2)^\mu \,\frac{i}{4} \quad = \frac{-2 p_2 \cdot p_4\, i}{v^2}$$

$$= i\frac{q^2}{v^2}$$

but $q^2 = (p_1 - p_3)^2$

$= (p_2 - p_4)^2$

$= 0 + 0 - 2p_2 \cdot p_4$

Others int by part

$$\mathcal{L} = \frac{1}{f^2}(\pi^0 \partial_\mu \pi^0)(\pi^+ \partial_\mu \pi^- + \pi^- \partial^\mu \pi^+)$$

$$= -\frac{1}{f^2} \pi^0 \pi^0 (\partial_\mu \pi^+ \partial^\mu \pi^-)$$

✓