Basics!

1/19/10

Note Title 1/19/2010

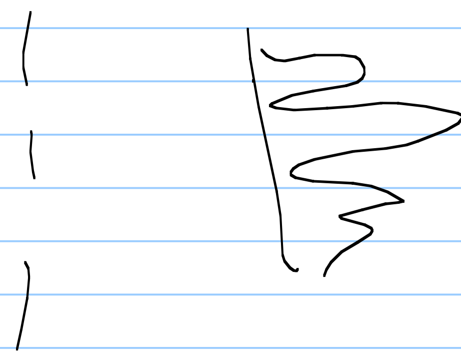
1905 – Quantum

- field → particle
- annihilated

Particles → fields

created

$e^+e^- \to \mu^+\mu^-$



QFT completion of QM

Course

- basics
- for all areas
    - rel. notation

Books -

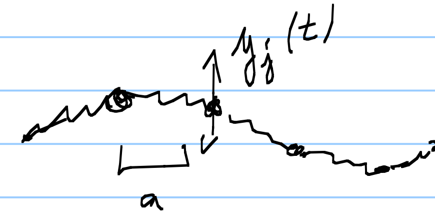
H.W.

Recording

Office hours - all , Google calendar

Construct a field — $\phi(x,t)$

$y_j(t)$



$$L = \sum_j \left[\frac{1}{2} m \dot{y}_j^2 - \frac{1}{2} k (y_{j+1} - y_j)^2 \right] = L(y_j, \dot{y}_j)$$

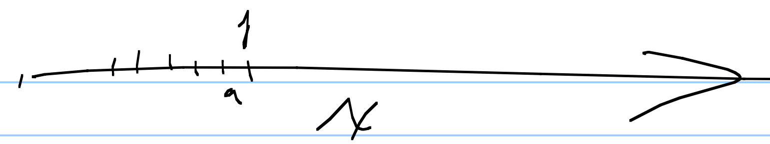
$$S = \int dt\, L$$

Classically

$$p_j = \frac{\partial L}{\partial \dot{y}_j} = m \dot{y}_j$$

$$H = \sum_j p_j \dot{y}_j - L = H(p_j, y_j) = \sum_j \left[\frac{p_j^2}{2m} + \frac{1}{2} k (y_{j+1} - y_j)^2 \right]$$

Continuum



$x = a j$

$\sum_j a = \int dx$

$y_j(t) = \frac{1}{\sqrt{ak}} \phi(x,t)$

$$\sum_j \frac{1}{2} k a^2 \left(\frac{y_{j+1} - y_j}{a}\right)^2 = \sum_j a \frac{1}{a} \frac{1}{2} k a^2 \frac{1}{ka} \left(\frac{\partial \phi}{\partial x}\right)^2 = \int dx \, \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2$$

$$\sum_j \frac{1}{2} m \dot{y}_j^2 = \int dx \frac{1}{2} \frac{1}{v^2} \left(\frac{\partial \phi}{\partial t}\right)^2 \quad , \quad v = \sqrt{\frac{ka^2}{m}}$$

$$** \quad S = \int dt\, dx \left[\frac{1}{2} \frac{1}{v^2} \left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 \right] = \int dt\, dx\, \mathcal{L}\left(\phi, \dot{\phi}, \frac{\partial}{\partial x}\phi\right)$$

↑ density

$L = \int dx \, \mathcal{L}$

Euler Lagrange eq.

$$\phi(x,t) = \bar{\Phi}(x,t) + \delta\phi(x,t)$$ ← $\delta\phi(x,t) = 0$ on any boundaries at infinity

↑ classical solution

$$\delta S = 0 = \int dt\,dx \left[\frac{1}{v^2} \frac{\partial \bar{\Phi}}{\partial t} \frac{\partial \delta\phi}{\partial t} - \frac{\partial \bar{\Phi}}{\partial x} \frac{\partial \delta\phi}{\partial x} \right]$$

↑ PLA

$$= \int dt\,dx \left[-\frac{1}{v^2} \frac{\partial^2 \bar{\Phi}}{\partial t^2} + \frac{\partial^2 \bar{\Phi}}{\partial x^2} \right] \delta\phi(x,t)$$

↑ int by parts

for any $\delta\phi(x,t)$

$$\left[\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right] \bar{\Phi}(x,t) = 0$$ wave eq.

Hamiltonian

$$p_j = \frac{\partial L}{\partial \dot{y}_j(t)} \qquad \Longrightarrow \qquad \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x,t)} \equiv \pi(x,t) = \frac{1}{v^2}\frac{\partial \phi}{\partial t}$$

$$H = \sum_j \left[\frac{p_j^2}{2m} + \frac{1}{2}k\left(y_{j+1} - y_j\right)^2\right] = \int dx \underbrace{\left[\underset{\uparrow\ \frac{1}{2}\pi^2 v^2}{\frac{1}{2v^2}\left(\frac{\partial\phi}{\partial t}\right)^2} + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2\right]}_{\mathcal{H}}$$

Field theory

$$H = \int dx\, \mathcal{H}$$

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \left[\frac{1}{2}\pi^2 v^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2\right] = \mathcal{H}(\pi, \phi)$$

Normalization

$$\frac{1}{v^2} = \frac{m}{ka^2}$$

$$p_j = m\dot{y}_j = \frac{m}{\sqrt{ka}}\,\dot{\phi}(x,t) = \frac{1}{v^2}\,a\sqrt{ka}\,\dot{\phi} = a\sqrt{ka}\;\Pi(x,t)$$

$$y_j(t) = \frac{1}{\sqrt{ka}}\,\phi(x,t)$$

Quantization

$$[y_j, p_{j'}] = i\hbar\, \delta_{j,j'}$$

$$= \left[\frac{1}{\sqrt{ka}} \phi(x,t),\ a\sqrt{ka}\, \pi(x',t)\right]$$

$$\boxed{[\phi(x,t), \pi(x',t)] = i\hbar \frac{\delta_{j,j'}}{a} = i\hbar\, \delta(x-x')}$$

← Equal time commutation rule

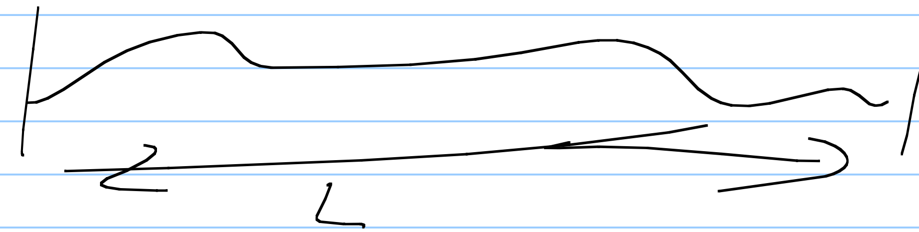
Correspondence

$$\sum_{j'} \delta_{j,j'} = 1 = \left(\sum_{j'} a\right) \underbrace{\frac{\delta_{jj'}}{a}}_{\delta(x-x') = \lim_{a\to 0} \frac{\delta_{jj'}}{a}} = \int dx'\, \delta(x-x') = 1$$

$x = ja$

Quanta – discrete notations

- Box normalization
- BC not important
- energy levels discrete



Solutions

$$\left[\frac{1}{v^2}\frac{\partial^2}{\partial t} - \frac{\partial^2}{\partial x^2}\right]\phi(x,t) = 0 \qquad \phi = N\left[e^{-i(\omega_s t - k_s x)} + c.c.\right]$$

$$\frac{\omega^2}{v^2} - k^2 = 0 \quad \Rightarrow \quad \omega_s = k_s v$$

General solution

$$\phi(x,t) = \sum_s N_s \left(e^{-i(\omega_s t - k_s x)} a(s) + e^{i(\omega_s t - k_s x)} a^{\dagger}(s) \right)$$

Conditions

$$[a(s), a(s')] = 0$$

$$[a(s), a^{+}(s')] = \delta_{ss'}$$

$$N_s = \sqrt{\frac{\hbar N^2}{2\omega_s L}}$$

$$\Pi(x,t) = \frac{1}{N^2}\frac{\partial\phi}{\partial t} = \sum_{s'} N_{s'} \frac{-i\omega_{s'}}{N^2}\left(e^{-i(\omega_{s'}t - k_{s'}x)} a(s') - e^{+i(\omega_{s'}t - k_{s'}x)} a^{+}(s')\right)$$

Check:

$$[\phi(x,t), \pi(x',t)] = \sum_{s,s'} N_s N_{s'}\left(\frac{-i\omega_{s'}}{N^2}\right)\left[\left(e^{-i(\ )} a(s) + e^{+i(\ )} a^{+}(s)\right), \left(e^{-i(\ )'} a(s') - a^{+}(s') e^{+i(\ )'}\right)\right]$$

$$= \sum_s \underbrace{N_s^2\left(\frac{+i\omega_s}{N^2}\right)}_{\frac{i\hbar}{L}} 2\ e^{ik_s(x-x')} = i\hbar \underbrace{\sum_s \frac{1}{L} e^{ik_s(x-x')}}_{\delta(x-x')}$$

$$= i\hbar\,\delta(x-x') \ \checkmark$$

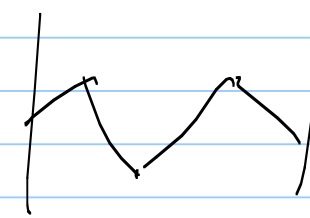
Hamiltonian

$$H = \int dx \; \frac{1}{2}\left[\frac{1}{v^2}\left(\frac{\partial \phi}{\partial t}\right)^2 + \left(\frac{\partial \phi}{\partial x}\right)^2\right]$$

$$\vdots$$

$$= \sum_s \hbar \omega_s \left(\underbrace{a^{+}(s)\, a(s)}_{\#\ \text{operator}} + \frac{1}{2} \right)$$

$\nwarrow E_0$



$$E = n \hbar \omega_s$$ $\Leftarrow$ <u>quanta</u>