Content of field theory notes and videos

Basic ideas

Here we directly construct a quantum field by hand. We go through the Lagrangian to define the Hamiltionian and the equation of motion. The quantization procedure is derived from that of quantum mechanics and we end up with quanta and the idea of a quantum field, as well as a representation of the energy states

Basics 1 – constructing a field, mass points, continuum limit, Lagrangian for the field, the wave equation, quantum commutation rules, solving with creation operators, quanta.

Basics 2– Solving the Hamiltonian, States and quanta, Continuous momentum notation, the Quantum Field, taking matrix elements, intuition

Basics 3 – canonical quantization, general equations of motion, more general scalar field, why phonons are massless, Heisenberg picture, why we use equal time commutators.

Basics 4 - equivalence with normal modes, inverting to get creation operators, zero-point energy, dimensional analysis

Introducing the fields

Here we go through the common fields used in field theory. Each produces slightly novel aspects in the quantization routine. At the end we know all of the fields as well as their propagators

Introducing Fields 1 – the common fields, the real scalar, the Feynman propagator, placing i epsilon in the right place.

Introducing fields 2 – the complex scalar, conserved currents, antiparticles, the non-relativistic field bosons and fermions.

Introducing Fields 3 – the Dirac field, solutions, quantization, conserved current, interpretation, Feynman propagator; Photons, Lagrangian and Hamiltonian.

Introducing fields 4 – quantization of photon, propagator, the Coulomb interaction

Interactions

Here we introduce interactions between the fields and go through the features of reading a full Lagrangian to uncover the key features. We identify the symmetries of a theory and the conserved quantities. We take matrix elements of the interaction Lagrangian. Perturbation theory is developed ending up at the Feynman rules

Interactions 1 – fields in electromagnetism, covariant derivatives and gauge invariance, the electromagnetic current

Interactions 2 - reading the Lagrangian, rules for taking matrix elements, crossing, introduction to Noether's theorem.

Interactions 3 – Conservation of energy and momentum. the energy-momentum tensor, internal symmetries, calculating the Noether current for particle number conservation, symmetries and interactions.

Interactions 4 – Perturbation theory, the time development operator, a scattering matrix element, interaction with a propagator, connection to old-style perturbation theory, variations on second example, Wick's theorem.

Interactions 5 – Loop diagrams, example - φ^4 to one loop, dropping disconnected diagrams, Feynman rules

Calculating in field theory

Here we show how the work that we have done can be used to form real observables – decay rates, cross sections, ground state energies and masses of the excitations. The latter topics lead us through spontaneous symmetry breaking, Goldstone's theorem and the Higgs mechanism

Calculating 1 – Decay rates and cross sections, Fermi's golden rule, Generalizing decay rates, Generalizing cross sections, example – scattering in φ^4

Calculating 2 – Review, comparison with delta function potential, short range interactions in field theory, Electromagnetic scattering, comparison with non-relativistic Coulomb scattering, Ground state energies and masses, symmetry breaking and Goldstone's theorem.

Calculating 3 – Using different names for fields, names don't count – Haag's theorem, Mandelstam variables s,t,u , Example – two scattering amplitudes. The Higgs mechanism, identifying the physical fields, mass generation, Status report.

<u>Introduction to renormalization</u>

When we calculate at higher orders in perturbation theory we have to think more carefully about the definitions of the parameters and their measurement. We discuss the ideas of renormalization and then give the BPH renormalization procedure for mass, charge and wavefunction renormalization. Φ^4 theory is treated completely. Because many of the loop integrals are divergent, we discuss the Pauli Villars and dimensional regularization schemes. Along the way we learn how to calculate loop integrals..

Renormalization 1 – Measuring the electron charge – charge renormalization – Poles in the propagator – mass renormalization. On-shell/ off-shell meanings

Renormalization 2 – review of ideas for mass and charge renormalization, dropping self energy diagrams, intro to formal technique for mass renormalization, wavefunction renormalization, complete renormalization of φ^4 theory, scattering example worked out, renormalization at threshold, renormalization at an unphysical point.

Renormalization 3 – Comments on BPHZ and conventional renormalizations, doing loop integrals, Feynman parameterization, Wick rotation, form of results and kinematic regions, origin of Imaginary part in amplitudes, unitarity.

Renormalization 4 – Infinities, Why do quantum calculations work? Philosophy of infinities, Regularization, Pauli-Villars regularization, dimensional regularization.

Renormalization 5 – finish dimensional regularization, renormalizable and "non-renormalizable" theories, MS and MS-bar renormalization schemes

<u>Overview of QED</u>

We tour the theory of QED – the QFT equivalent of the role that the hydrogen atom plays in QM. We first describe the renormalization of the theory, and then look at some of the predictions. The topics covered include the Lamb shift, running coupling constant, g-2, and bremsstrahlung.

QED 1 – Overview of QED, charge quantization , gauge invariance condition, mass and wavefunction renormalization, photon wavefunction renormalization, vertex function, charge renormalization, role of Ward identity

QED 2 – The Lamb shift, running coupling constant and the renormalization group, infrared divergences, bremsstrahlung, g-2.

Path Integrals and Functional Methods

Here we develop QM and QFT from a path integral perspective, with a focus on functional methods. We define the generating functional and show how this yields matrix elements in quantum mechanics. Then this technique is used to show how one obtains the Feynman rules via path integral methods. The path integral formalism is closely related to quantum statistical mechanics, and this correspondence is displayed. We close with a discussion of the idea of "integrating out" high energy degrees of freedom, and the connection to the Wilsonian approach to the renormalization group

Functional 1: Motivation for path integrals, some Gaussian integrals, Derivation of path integrals in quantum mechanics, content of the path integral, projecting out the ground state

Functional 2 – Classical limit, functional differentiation, generating functional, Harmonic oscillator matrix elements, QFT path integration, Field theory generating functional and matrix elements.

Functional 3 – Review of generating functional, interpretation of J as source, Free field path integral, two-point function, four-point function, interactions, evaluating Z[J] in perturbation theory, two-point function to order lambda

Functional 4 – The four-point function, the rest of perturbation theory, the connection of the path integral with classical statistical mechanics, finite temperature propagator, integrating out fields

Functional 5 – Integrating out fields with a Gaussian integral, connection to Wilsonian renormalization group, LSZ reduction

Homework exercises

1) Energy conservation
2) Eigenfunction expansion of a field
3) The Landau theory of phase transitions as a field theory
4) The Higgs mechanism and the Meissner effect as a field theory
5) Practice reading Lagrangians
6) Practice finding Noether current
7) A decay rate – from start to finish
8) A cross section
9) Drawing loop diagrams
10) Introduction to the renormalization group
11) The Ward identity
12) The harmonic oscillator via functional methods
13) The magnetic correlation function and critical exponents