1. Compute the fraction $\Delta t/\tau$ of a complete period $\tau = 2\pi/\omega_0$ that a simple harmonic oscillator with amplitude $A$ spends within a small given interval $\Delta x$ at a position $x$. Discuss the result.

2. Suppose that the amplitude of a damped oscillator becomes equal to $(1/e)$ times its initial value after $n$ periods. Show that the frequency of the oscillator must be approximately $[1 - (8\pi^2n^2)^{-1}]^n$ times the frequency of the corresponding undamped oscillator.

3. A particle of mass $m$ is at rest at the end of a spring with a force constant $k$. The spring is hanging from a fixed support. At $t = 0$, a constant downward force $F$ is applied to the mass and acts for a time $t_0$. Show that, after the force is removed, the displacement of the mass from its equilibrium position ($x = x_0$ where $x$ is down) is $x - x_0 = (F/k) [\cos \omega_0(t - t_0) - \cos \omega_0t]$, where $\omega_0 = (k/m)^{1/2}$.

4. A particle is under the influence of a force $F = -kx + kx^3/\alpha^2$, where $k$ and $\alpha$ are constants and $k > 0$. Find the corresponding potential $U(x)$ and discuss the motion. What happens when $E = k\alpha^2/4$?

5. A particle of mass $m$ moving in one dimension has potential energy $U(x) = U_0 [2(x/a)^2 - (x/a)^4]$, where $U_0$ and $a$ are positive constants. (a) Find the force $F(x)$, which acts on the particle. (b) Sketch $U(x)$. Find the position of stable and unstable equilibrium (Note: Equilibrium point corresponds to the solution of $(dU/dt)_{t=0} = 0$. If $(d^2U/dt^2)_{t=0} > 0$, the equilibrium is stable, if $(d^2U/dt^2)_{t=0} < 0$, the equilibrium is unstable). (c) What is the angular frequency $\omega$ of oscillations about the point of stable equilibrium? (d) What is the minimum speed the particle must have at the origin to escape to infinity? (e) At $t = 0$ the particle is at the origin and its velocity is positive and equal in magnitude to the escape speed of part ”(d)”. Find $x(t)$ and sketch the result.