Homework 7
Due Thursday the 2 of November

HW problem 1: Screened potential. A particle moves in a central potential field $U(r) = -\frac{ke^{-r\kappa}}{r}$ (screened potential). For what values of the angular momentum $l_0$ a bounded motion of the particle (i.e. when the particle does not escape to the infinity) is possible?

Hints:
1. A bounded motion is possible if the effective potential has a minimum. It has a minimum if the following equation has a solution: $dV_{eff}(r)/dr = 0$.
2. Introduce a dimensionless variable $x = \kappa r$.
3. The function $f(x) = x(x + 1)e^{-x}$ at $x \geq 0$ starts at $f(0) = 0$, then upon increasing $x$ reaches a maximum at some value of $x$, after which decreases exponentially. The maximal value of $f(x)$ is $f_{\text{max}} = (2 + \sqrt{5}) \exp\left(-\frac{1+\sqrt{5}}{2}\right) \approx 0.84$. Analyze this function analytically, find $f_{\text{max}}$. Check if the value above is correct.
4. Notice that the bounded motion is possible if $f(x)$ is smaller than an $l_0$-dependent constant.

HW problem 2: Rutherford scattering. Find the minimal distance between particles, when one of them (having mass $m_1$) is coming from infinity with initial velocity $v$, and approaching an initially resting particle (having mass $m_2$) with impact parameter $\rho$ (the impact parameter is the perpendicular distance between the initial path of an approaching particle and the target particle, see e.g. https://en.wikipedia.org/wiki/Impact_parameter). These particles interact repulsively with via central potential $U(r) = k/r^n$.

Hints:
1. The relative motion is characterized by the conserved angular momentum $l = mvp$, and energy $E = mv^2/2$, where $m$ is the reduced mass.
2. The minimal distance between particles, $r_{\text{min}}$, is defined by the condition $V_{eff}(r_{\text{min}}) = E$. 