Introduction
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Outline and prerequisites: See Syllabus

Books and additional material: See Syllabus as well. These lecture notes are far from being original, and were inspired by other lectures I myself attended, on other sets of lecture notes of friends and colleagues (some of which are available online): J. McGreevy, H. Saleur, A. Vishwanath, S. Parameswaran and A. Potter.

Overview:

Lattice model (ex: spins, interacting electrons) \( \Rightarrow \) QFT (quantum field theory) low energy long distances

Examples we will encounter:

- \( S = \frac{1}{2} \int d^2 \bar{z} \left[ \bar{X}_+ \partial \bar{X}_+ + \bar{X}_- \partial \bar{X}_- \right] \) Majorana / Ising CFT = Describing the critical point in \( Z_2 \) systems in 1d

- \( H = - i v_F \int dx \left[ \bar{\psi}_R \gamma_x \psi_R - \bar{\psi}_L \gamma_x \psi_L \right] \) Massless Dirac Fermion in 1d

= low energy QFT describing the spin 1/2 XX chain:

\[ H = J \sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) \]
\[ H = \frac{\nu}{2\pi} \int dx \left[ k (\partial_x \phi)^2 + \kappa^{-1} (\partial_x \theta)^2 \right] \]

Luttinger liquid (= Free Boson CFT)

\[ = \text{Describes the generic low energy physics of interacting } e_- \text{ in } 1d \quad (= \text{non-Fermi liquid}) \]

\[ Z_2 \text{ and } U(1) \text{ gauge theories in } 2+1d \quad (2d \text{ quantum system}) \]

**Some concepts:**

- Quantum (states/operators) \(\leftrightarrow\) Classical (path integral, partition function)

- Thermal partition function = Propagation in imaginary time

- Objects of interest: correlation functions, critical exponents

- Universality: long-distance \& low-energy properties independent of microscopic details (ex: critical exponents, topological properties etc.).

  \(\Lambda\)s depend only on global structures: dimensionality, symmetry, topological defects...

**Emergence:** Lorentz symmetry, conformal symmetry, SUSY, gauge invariance, fractional charge and statistics can emerge even if absent at the microscopic level.

\[ \text{ex} \quad \begin{array}{c}
\Lambda \\
\text{spinon}
\end{array} \quad 1d \text{ interacting wire} \]

\[ e^- \quad \Rightarrow \quad \text{spin} \end{center} \quad \text{charge separation} \]
\[ \begin{array}{c}
\text{Objects of interest:} \\
\text{Groundstate: (GS)} \quad \text{“vacuum” + low energy excitations} \\
\rho = \frac{1}{Z} e^{-\beta H} \quad \text{(thermal physics)} \\
\text{Dynamics: } U(t) = e^{iHt} \quad (t = i\beta \text{: thermal physics}) \\
\text{Typically: compute } U(\tau) = e^{-\tau H} \text{ then } \tau = i t \quad (\text{Wick notation})
\end{array} \]
Correlation function: \[ \langle \sigma(x, t) \sigma(x', t') \rangle \]

\[ \langle \cdots \rangle_T = \frac{1}{Z} \text{Tr} \left( e^{-\beta H} \cdots \right) , \quad \text{at } T = 0 \quad \langle \cdots \rangle = \langle \text{GS} \cdots \text{GS} \rangle \]

e.g. magnetic system: \[ \langle m(x) m(0) \rangle \sim \int \frac{d^D x}{x^{2a}} \left\{ \begin{array}{ll} \text{cst} & \text{Ferromagnet} \\ e^{-x/\xi} & \text{Para magnet} \end{array} \right\} \]

Cutoffs: We'll see that dealing with QFTs leads to many "infinities" that we'll have to deal with. (e.g.: GS energy!)

QFT = effective description valid at length-scales \( \rho \gg a \)

\[ \uparrow \text{UV cutoff, lattice spacing} \]

In particle physics: Planck scale \( \rho_p \sim 10^{-35} \text{ m} \)

Lattice models = "UV regularizations" of QFTs

Not all QFTs can be put on a lattice: (e.g. "Anomalies"
"UV complete")

\[ \text{eg chiral fermion } H = -i \gamma^\mu \int dx \gamma^\mu \partial_\mu \psi \]

\[ \text{and single} \]

\[ \text{caution! Fermion } H = -i \int dx \gamma^\mu \partial_\mu \psi \]