Turn off all cell phones and watch alarms! Put away iPods, etc.

You may use a calculator. If you do, be sure to show the set-up for what you are calculating and do not round intermediate results.

Otherwise, this is a “closed-book” exam: do not use any reference materials or paper other than this exam booklet.

Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.

Organize your work in an unambiguous order. Show all necessary steps.

Answers given without supporting work may receive 0 credit!

Be ready to show your UMass ID card when you hand in your exam booklet.
1. (16%) A car is traveling north on a highway into a snowstorm. Due to steadily worsening conditions, the velocity (in miles per hour) of the car at time $t$ hours after the trip begins is given by

$$v(t) = 60e^{-2t}$$

(a) Write a definite integral that represents the distance the car travels during the first three hours of the trip.

(b) Evaluate the integral in part (a).
2. (16%)
Consider the region to the right of $x = 1$, between $y = 0$ and $y = \frac{1}{x}$.

(a) Set up and evaluate an improper integral to determine whether the region has finite area. If the area of the region is finite, determine what that area is. If it is not finite, show it.

(b) Consider the solid formed by rotating that region around the $x$-axis. Set up and evaluate an improper integral to determine whether the volume of the solid is finite, and if the volume is finite, determine its value.
3. (10%) Determine whether the series

\[ \sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n^5}. \]

is absolutely convergent, conditionally convergent, or divergent. Cite the test(s) you use and show how they apply here.
4. (10%) Find the Maclaurin series for \( f(x) = \ln(1 + 3x) \). Express your answer in \( \Sigma \) notation.
5. (16%)

Recall that the Maclaurin series for $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n + 1)!}$.

(a) Use the Maclaurin series for $\sin(x)$ to find a power series for $\sin(2x^3)$.

(b) Find the interval of convergence for the series you found in part (a).
(Justify your answer.)
Continuation of # 5.

(c) Use the series you found in part (a) to compute the limit

\[ \lim_{x \to 0} \frac{\sin(2x^3)}{x^3} \]
6. (16%) As the parameter $t$ increases from $t = 0$, the curve with parametric equations

$$x = e^{-t} \cos t, \quad y = e^{-t} \sin t$$

spirals inward towards the origin as $t \to \infty$.

(a) Set up and evaluate an improper integral that gives the length of the entire curve.

(b) Find the slope of the tangent line to the curve at the point where $t = \pi/2$. 
7. (16%) A cardioid is given by the polar equation \( r = 1 + \cos(\theta) \).

(a) Shade the region bounded by the cardioid \( r = 1 + \cos(\theta) \), \( \theta = 0 \) and \( \theta = \pi/2 \).

(b) Set up a definite integral that represents the area of the shaded region in (a).

(c) Evaluate the definite integral in (b).
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