No calculators, papers, or notes may be used.

Please don’t just give an answer. Clearly explain how you get it, providing appropriate mathematical details. An answer with no supporting work will be awarded zero points.

This is a 2 hour exam.

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<td>MC Total (Out of 25)</td>
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<td>6 (Out of 20)</td>
<td>a.</td>
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<td>7 (Out of 15)</td>
<td>a.</td>
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<td>8 (Out of 20)</td>
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<td>9 (Out of 20)</td>
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Multiple Choice Section: Choose the one option that best answers the question. There is no partial credit for questions 1-5.

1. [5 points] Find a polar equation for the curve represented by the Cartesian equation \( x = 6 \).
   (A.) \( r = 6 \tan \theta \)
   (B.) \( r = 6 \)
   (C.) \( r = 6 \cos \theta \)
   (D.) \( r = 6 \sec \theta \)

2. [5 points] Which of the following is the radius of convergence for the power series?

\[
\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^{25} n}
\]

   (A) \( \frac{1}{5} \)
   (B) 1
   (C) 5
   (D) \( \infty \)

3. [5 points] For which \( x \) values does the following series converge?

\[
\sum_{n=1}^{\infty} \frac{x^{n-1}}{3^n}
\]

   (A) \(-3 < x < 3\)
   (B) \(-\frac{1}{3} < x < \frac{1}{3}\)
   (C) \(-3 \leq x \leq 3\)
   (D) \(-\frac{1}{3} \leq x \leq \frac{1}{3}\)
4. [5 points] Consider the integral \( \int \ln(2x) \, dx \). Which of the following is true?
   (A) This integral does not exist.
   (B) This integral can be found using integration by parts.
   (C) This integral can be found using u-substitution.
   (D) This integral can be found using trigonometric substitution.

5. [5 points] Which of the following is a Polar representation of the Cartesian coordinate \((\sqrt{3}, -1)\)?
   (A) \( (2, -\frac{\pi}{3}) \)
   (B) \( (2, \frac{5\pi}{6}) \)
   (C) \( (2, \frac{5\pi}{3}) \)
   (D) \( (2, -\frac{\pi}{6}) \)

Please fill in your letter answer for questions 1-5 below:

(1) _______ (2) _______ (3) _______ (4) _______ (5) _______
**Free Response Portion:** Show all work for each of the following questions. Partial credit may be awarded for questions 6-9.

6 (a). [10 points] Evaluate the integral.

\[ \int_{0}^{1} x^3(1 + x^4)^4 \, dx \]
6 (b). [10 points] Represent the following function as a power series. Express your answer in summation notation and simplify completely within the summation.

\[ f(x) = \frac{x^2}{(1 + 4x^3)^2} \]
7 (a). [10 points] Find the Taylor series for the function below. Express your answer in summation notation and simplify completely within the summation.

\[ f(x) = \ln(1 + x) \]

Centered at \( a = 1 \)
7 (b). [5 points] The Maclaurin series of $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n + 1}$.

Determine the Maclaurin series for

$$f(x) = 9x \tan^{-1}(4x^3)$$

Express your answer in summation notation and simplify completely within the summation.
8 (a). [10 points] Find the radius and interval of convergence of the series. Justify any test you use, and be sure to verify any necessary conditions.

\[ \sum_{n=0}^{\infty} (-1)^n \frac{(5x)^n}{3\sqrt{n} + 2} \]
8 (b). [10 points] Consider the polar equation \( r = \cos(5\theta) \) given in the graph below.

Calculate the area enclosed in one loop of the curve. Mathematically justify how you find the integral bounds.
9 (a). [10 points] Find the exact length of the parametric curve below.

\[
x = \frac{1}{2} t^2
\]

\[
y = \frac{1}{3} (2t + 1)^{3/2}
\]

\[0 \leq t \leq 4\]
9 (b). [10 points] Find the equation of the line tangent to the parametric curve given below at the given point. Express your answer as $y = f(x)$.

\[ x = \sec(t) \]

\[ y = \tan(t) \]

\[ t = \frac{\pi}{6} \]
This page is intentionally left blank for additional work.