• No papers or notes may be used. You may use a calculator on this exam.

• Please don’t just give an answer. Clearly explain how you get it, providing appropriate mathematical details. An answer of ‘convergent’ or ‘divergent’ with no supporting work will be awarded zero points.

• This is a 2 hour exam.

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<td>6 (Out of 20)</td>
<td>a.</td>
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<td>7 (Out of 15)</td>
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<td>8 (Out of 20)</td>
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<td>9 (Out of 20)</td>
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Multiple Choice Section: Choose the one option that best answers the question. There is no partial credit for questions 1-5.

1. [5 points] For what values of \( r \) does the following integral converge?

\[
\int_{r}^{\infty} \frac{1}{x^2} \, dx
\]

(A) For all values of \( r \).
(B) \( r > 0 \).
(C) \( r \geq 0 \).
(D) Never.

2. [5 points] Which one of the following series represents the repeating decimal \( 0.454545... \)?

(A) \[ \sum_{n=1}^{\infty} 45 \left( \frac{1}{10} \right)^{n-1} \]
(B) \[ \sum_{n=1}^{\infty} \frac{45}{100} \left( \frac{1}{100} \right)^{n-1} \]
(C) \[ \sum_{n=1}^{\infty} 45 \left( \frac{1}{100} \right)^{n-1} \]
(D) \[ \sum_{n=1}^{\infty} \frac{45}{100} \left( \frac{1}{100} \right)^{n} \]

3. [5 points] Which of the following statements is false?

(A) If the series \( \sum_{n=1}^{\infty} a_n \) converges, then \( \lim_{n \to \infty} a_n = 0 \).

(B) If \( \lim_{n \to \infty} a_n = 0 \), then the series \( \sum_{n=1}^{\infty} a_n \) converges.

(C) If \( \lim_{n \to \infty} a_n \neq 0 \), then the series \( \sum_{n=1}^{\infty} a_n \) diverges.

(D) If \( \lim_{n \to \infty} a_n = 1 \), then the sequence \( \{a_n\}_{n=1}^{\infty} \) converges.
4. [5 points] Given the following series, which of the following converge?

I. $\sum_{n=1}^{\infty} \frac{2n}{n^3}$

II. $\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}}$

III. $\sum_{n=1}^{\infty} \frac{n + 8}{14n + 9}$

(A) I, II, and III
(B) I and II
(C) I only
(D) II and III

5. [5 points] What is the sum of the series $\sum_{n=0}^{\infty} \frac{3^{n-1}}{5^n}$?

(A) $\frac{3}{5}$

(B) $\frac{1}{2}$

(C) $\frac{5}{6}$

(D) 1

Please fill in your letter answer for questions 1-5 below:

(1) _______ (2) _______ (3) _______ (4) _______ (5) _______
Free Response Portion: Show all work for each of the following questions. Partial credit may be awarded for questions 6-10. You will receive no credit for an answer of “convergent” or “divergent” without supporting work.

6 (a). [10 points] Evaluate the following integral.

\[
\int \frac{x + 1}{2x^2 + x - 3} \, dx
\]
6 (b). [10 points] Does the following integral converge or diverge? If it converges, what does it converge to?

\[ \int_{2}^{3} \frac{3}{(x-2)^2} \, dx \]
7 (a). [5 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=2}^{\infty} \frac{\sin^2 n}{n^2 + 1} \]
7 (b). [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=2}^{\infty} \frac{(n + 1)^n}{2^{n+1}(-\ln(n))^n} \]
8 (a). [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[
\sum_{n=1}^{\infty} 6n^2 e^{-n^3}
\]
8 (b). [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=2}^{\infty} \frac{\sqrt{n^2 - 2n + 3}}{n^3 + n + 1} \]
9 (a). [10 points] Does the series absolutely converge, conditionally converge, or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+3}5^n}{3^n(2n)!}$$
9 (b). [10 points] Does the series absolutely converge, conditionally con-
verge, or diverge? State which test you used and clearly show that the series
meets the conditions to use this test.

\[ \sum_{n=1}^{\infty} (-1)^n \frac{n + 1}{n^2 + 7} \]
This page is intentionally left blank for additional work.