Math 132 Final Exam Spring 2015

Name: ________________________________

ID Number: __________________________

Section Number: ______________________

Section  Day/Time  Instructor  Section  Day/Time  Instructor
1    MWF 10:10  Farelli  9    TuThu 1:00  Benincasa
2    MWF 9:05  Farelli  10   TuThu 2:30  Benincasa
3    MWF 11:15 Clark  11   MWF 10:10  Buskin
4    MWF 12:20 Clark  12   MWF 12:20  Yaping
5    MW 2:30  Brown  13   MWF 1:25  Yaping
6    MW 4:00  Brown  15   TuThu 11:30 Buckman
7    TuThu 8:30 Duanmu  16   TuThu 1:00  Wen
8    TuThu 10:00 Oloo  17   TuThu 2:30  Wen

• No calculators, papers, or notes may be used.

• Please don’t just give an answer. Clearly explain how you get it, providing appropriate mathematical details.

• This is a 2 hour exam.

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1
**Multiple Choice Section:** Choose the one option that best answers the question. There is no partial credit for questions 1-5.

1. [5 points] Find the Cartesian coordinate of the polar coordinate \((r, \theta) = (\sqrt{2}, 5\pi/4)\).
   
   (a) \((1, 1)\)
   
   (b) \((-1, -1)\)
   
   (c) \((1, -1)\)
   
   (d) \((-1, 1)\)

2. [5 points] Which is not a possible result for a power series?

   \[
   \sum_{n=0}^{\infty} c_n (x - a)^n
   \]

   (a) the series converges if \(x = a\)
   
   (b) the series converges for all \(x\)
   
   (c) the series converges when \(|x - a| < R\)

   (d) the series converges when \(|x - a| > R\)
3. [5 points] Find the interval of convergence of the power series:

\[ \sum_{n=1}^{\infty} n! (3x - 1)^n \]

(a) \( I = \{0\} \)

(b) \( I = (-\frac{1}{3}, \frac{1}{3}] \)

(c) \( I = \emptyset \)

(d) \( I = \left\{ \frac{1}{3} \right\} \)

4. [5 points] Eliminate the parameter to find the Cartesian equation for \( x = 5 \sin(t), y = 2 \cos(t) \).

(a) \( \frac{x^2}{4} + \frac{y^2}{25} = 1 \)

(b) \( \frac{x^2}{25} + \frac{y^2}{4} = 1 \)

(c) \( x^2 + y^2 = 1 \)

(d) \( x^2 + y^2 = 10 \)

5. [5 points] Convert the polar equation \( 3r \cos(\theta) + 4r \sin(\theta) = 1 \) into a Cartesian equation.

(a) \( 3x + 4y = x^2 + y^2 \)

(b) \( \frac{x}{3} + \frac{y}{4} = 1 \)

(c) \( 3x + 4y = 1 \)

(d) \( 3y + 4x = 1 \)

Please fill in your letter answer for questions 1-5 below:

(1) _______  (2) _______  (3) _______  (4) _______  (5) _______
Free Response Portion: Show all work for each of the following questions. Partial credit may be awarded for questions 6-9. You will receive no credit for an answer without supporting work.

6. (a) [5 points] Evaluate the integral.

\[ \int x^2 \sin(x^3) \, dx \]

\[ -\frac{1}{3} \cos(x^3) + C \]
(b) [5 points] The Maclaurin series of \( \sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n + 1)!} \).

Determine the Maclaurin series for

\[
f(x) = 2x^3 \sin(2\pi x^2) \]

Simplify completely by combining all terms within the summation.

\[
\sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+2} \pi^{2n+1} x^{4n+5}}{(2n + 1)!} \]
7. (a) [10 points] Find the interval of convergence of the series. Justify any test you use, and be sure to verify any necessary conditions.

\[ \sum_{n=0}^{\infty} (-1)^n \frac{(5x - 3)^n}{3n + 2} \]

\[ I = \left( \frac{2}{5}, \frac{4}{5} \right] \]
(b) [10 points] Represent the following function as a power series. Simplify completely by combining all terms within the summation.

\[
f(x) = \frac{x^6}{(1 - 4x)^2}
\]

\[
\sum_{n=0}^{\infty} 4^n (n + 1) x^{n+6}
\]
8. Consider the curve given by the parametric equations

\[ x = e^{\sin(t)} \quad y = \cos(t) + t - \pi \]

\[ 0 \leq t \leq 2\pi \]

(a) [10 points] Find the equation of the tangent line to the curve at the point \((1, -1)\).

\[ y = -x \]

(b) [10 points] Find all the points where there is a vertical tangent line on the interval \(0 \leq t \leq 2\pi\).

\[ (e, -\frac{\pi}{2}), \left(\frac{1}{e}, \frac{\pi}{2}\right) \]
9. (a) [10 points] Find the area enclosed in all loops of the function \( r = \cos(2\theta) \) given the graph of the function below. Mathematically justify how you find the integral bounds.

\[
A = \frac{\pi}{2}
\]
(b) [10 points] Find the slope of the tangent line for the function in part (a) \( r = \cos(2\theta) \) at \( \theta = \pi/4 \).

\[ m = 1 \]

(c) [5 points] Find the exact length of the curve \( r = e^{3\theta} \) from \( \theta = 0 \) to \( \theta = 2 \).

\[ \frac{\sqrt{10}}{3} (e^6 - 1) \]