• No papers or notes may be used. You may use a calculator on this exam.

• Please don’t just give an answer. Clearly explain how you get it, providing appropriate mathematical details. An answer of ‘convergent’ or ‘divergent’ with no supporting work will be awarded zero points

• This is a 2 hour exam.

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<tr>
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<tr>
<td>1</td>
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<td>Benincasa</td>
<td>TuThu 1:00</td>
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**Multiple Choice Section:** Choose the one option that best answers the question. There is no partial credit for questions 1-5.

1. [5 points] Consider the following:

\[ a_n = \frac{n^3}{3n^3 + 5} \]

I. \( a_n \) converges
II. \( a_n \) diverges
III. \( \sum_{n=1}^{\infty} a_n \) converges
IV. \( \sum_{n=1}^{\infty} a_n \) diverges

Which of the following is true?

(a) I & III
(b) I & IV
(c) II & III
(d) II & IV

2. [5 points] Which of the following are true for the Test for Divergence:

I. If \( \lim_{n \to \infty} a_n \) does not exist, then \( \sum_{n=0}^{\infty} a_n \) diverges.

II. If \( \lim_{n \to \infty} a_n \) exists but is not 0, then \( \sum_{n=0}^{\infty} a_n \) diverges.

III. If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=0}^{\infty} a_n \) converges.

(a) I & II
(b) I & III
(c) I, II & III
(d) II & III
3. [5 points] Consider two positive series \( \sum a_n \) and \( \sum b_n \) with \( a_n \leq b_n \). Which of the following is definitely true?

(a) If \( \sum a_n \) converges, then \( \sum b_n \) converges.
(b) If \( \sum b_n \) diverges, then \( \sum a_n \) diverges.
(c) If \( \sum b_n \) converges, then \( \sum a_n \) converges.
(d) If \( \lim_{n \to \infty} \frac{a_n}{b_n} = 0 \), then both \( \sum a_n \) and \( \sum b_n \) converge or both diverge.

4. [5 points] Determine if the following series converges, and if so, the value it converges to.

\[
\sum_{n=1}^{\infty} (-1)^n \frac{7 \cdot 4^n}{3^n}.
\]

(a) \(-21\)
(b) \(-4\)
(c) \(3\)
(d) divergent

5. [5 points] Which of the following yields an inconclusive result for the Root Test?

(a) \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = 1 \)
(b) \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = \infty \)
(c) \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L < 1 \)
(d) \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L > 1 \)

Please fill in your letter answer for questions 1-5 below:

(1) _____  (2) _____  (3) _____  (4) _____  (5) _____
**Free Response Portion:** Show all work for each of the following questions. Partial credit may be awarded for questions 6-9. You will receive no credit for an answer of “convergent” or “divergent” without supporting work.

6.

(a) [5 points] Evaluate the following integral.

\[ \int \frac{y}{y^2 - 2y - 3} \, dy \]

(b) [10 points] Does the following integral converge or diverge? If it converges, what does it converge to?

\[ \int_{2}^{3} \frac{1}{\sqrt{3-x}} \, dx \]
7.

(a) [10 points] Find the values of $x$ for which the series is convergent.

$$\sum_{n=0}^{\infty} (3)^{n+1}(x + 4)^n$$

(b) [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=1}^{\infty} \frac{\cos^2(n)}{5^n}$$
8.

(a) [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=1}^{\infty} \frac{4n}{6n^2 + 2n + 8} \]

(b) [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=3}^{\infty} \frac{\ln(n^2)}{n} \]
(a) [10 points] Does the series absolutely converge, conditionally converge, or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=1}^{\infty} \frac{n^3(n + 1)!}{e^n n!} \]
(b) [10 points] Does the series absolutely converge, conditionally converge, or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

\[ \sum_{n=2}^{\infty} (-1)^{n+1} \frac{5}{3\sqrt{n} - 3} \]
This page is intentionally left blank for additional work.