Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are ten (10) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- You *may* use a calculator. If you do, be sure to show the set-up for what you are calculating and do *not* round intermediate results.
- Otherwise, this is a “closed-book” exam: do not use any books or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

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1. (10%) Consider \( f(x) = \frac{x + 2}{x^2 - 4} \).

(a) By calculating relevant limits, determine the equations of all vertical asymptotes of the graph of \( f(x) \). If there are none, say so.

(b) By calculating relevant limits, determine the equations of all horizontal asymptotes of the graph of \( f(x) \). If there are none, say so.
2. (10%) Evaluate the following limits (Don’t use calculators or tables to compute limits).

(a) \( \lim_{x \to \infty} \frac{1 - x^3}{3x^3 + 2x - 7} \)

(b) \( \lim_{x \to \infty} \frac{\sqrt{9x^2 + 11}}{x + 2} \)
3. (10%) Evaluate the following limit (Don’t use calculators or tables to compute the limit).

\[ \lim_{x \to -1} \frac{\sqrt{x^2 + 3} - 2}{x + 1} \]

4. (10%) Evaluate the following limit (Don’t use calculators or tables to compute the limit).

\[ \lim_{x \to \infty} e^{-x} \cos x \]
5. (10%) Evaluate the following limit (Don’t use calculators or tables to compute the limit).

\[
\lim_{x \to -2} \frac{x^2 - 4}{(x + 2)(x^2 + 5)}
\]

6. (10%) Let \( f(x) = 1 - \sqrt{1 - x} \).

   (a) For which values of \( x \) is \( f(x) \) continuous?

   (b) Use the Intermediate Value Theorem to show that the equation

   \[
   1 - \sqrt{1 - x} = 0
   \]

   has a zero in the interval \((-\frac{1}{2}, \frac{1}{2})\).
7. (10%) Find the derivative of the function \( f(x) = 17 - \frac{x}{e} \) using the definition of the derivative. Do NOT use any rules of differentiation that you may have learned in a previous calculus class.

8. (10%) Let \( f(x) = 4x^2 - 2 \)

(a) According to the limit laws, what is the value of \( \lim_{x \to 3} f(x) = L \)?

(b) Using the precise definition of the limit, prove that \( \lim_{x \to 3} f(x) = L \) where \( L \) is the number you gave in part (a). Proceed as follows: for \( \varepsilon > 0 \) a given real number (but arbitrary), find an appropriate expression for a real number \( \delta \) in terms of \( \varepsilon \), such that \( |f(x) - L| < \varepsilon \) whenever \( 0 < |x - 3| < \delta \).
9. Let

\[ f(x) = \begin{cases} 
  x^2 + 1 & \text{if } x > 0 \\
  (x + 1)^2 & \text{if } x \leq 0 
\end{cases} \]

Show that \( f(x) \) is continuous at \( x = 0 \). You must argue this algebraically, without using a table or graph.

10. (10%) Let \( f(x) = \frac{x}{x+1} \). The derivative of \( f \) is given by \( f'(x) = \frac{1}{(x+1)^2} \).

Find an **equation** of the tangent line to the graph of \( f \) at the point where \( x = 1 \).
This page left blank for additional work.