# Holographic Superconductors

"Building a Holographic Superconductor, " Hartnoll, Herzog & Horowitz, PRL 101 (2008) 031601, arXiv:0803.3295

"Breaking an Abelian gauge symmetry group near a black hole horizon," Gubser, PRD 78 (2008) 065034, arXiv:0801.2977

## Holographic duals

QFT in D dimensions ⟺ Quantum gravity system in D+1 dimensions

Originally motivated by black hole entropy....

The quantum physics of the black hole interior is captured by information on the event horizon.

Maldacena, 1997

Realized most explicitly in AdS/CFT correspondence of string theory.

Type IIB string theory on $AdS_5 \times S^5$ with N units of 5-form flux on the $S^5$ ⟺ Maximally supersymmetric SU(N) Yang-Mills theory

Strong coupling limit of gauge theory well described by classical (super)gravity

Holographic Superconductors → "Applied AdS/CFT"

See whether AdS/CFT strategy can be useful more broadly beyond string theory context

Quark gluon plasma, other CM systems

Adopt a more phenomenological, model building attitude. →

Construct a gravitating system with a set of desired properties

Infer results for dual strongly coupled QFT

For superconductivity want….

Standard superconductors well described by weakly coupled BCS theory.

- Phase transition with low temperature, charged scalar condensate
- infinite DC conductivity in low T phase
- Also look at magnetic properties (Meissner effect), frequency dependent conductivity (energy gap)

Hope to learn about mechanism for high-$T_C$

Evidence of strongly coupled electron pairing

Superconductivity concentrated on 2D planes

# Introduction to AdS/CFT

deSitter – Spacetime of constant positive curvature $\Lambda > 0$ cosmology

Anti-deSitter – Spacetime of constant negative curvature $\Lambda < 0$ supersymmetry

Spacetime analogues of spheres and hyperboloids

$$L^2 = x_1^2 + \cdots + x_{n+1}^2$$

$$-L^2 = -x_1^2 + x_2^2 + \cdots + x_{n+1}^2$$

dS$^n$ $$L^2 = -x_0^2 + x_1^2 + \cdots + x_{n+1}^2$$

AdS$^n$ $$-L^2 = -x_0^2 - x_1^2 + x_2^2 + \cdots + x_{n+1}^2$$

↑

Curvature radius $$L^2 = -\frac{3}{\Lambda}$$ For n=4

## AdS acts like a box

Work in static coordinates in D=4….

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

| | | |
|---|---|---|
| $f = 1 - \frac{2M}{r}$ | Schwarzschild BH | $\Phi = -\frac{M}{r}$ |
| $f = 1 - \frac{r^2}{L^2}$ | deSitter | $\Phi = -\frac{r^2}{2L^2}$ |
| $f = 1 + \frac{r^2}{L^2}$ | Anti-deSitter | $\Phi = +\frac{r^2}{2L^2}$ |

Newtonian potential $\Phi = -\frac{1 + g_{tt}}{2}$

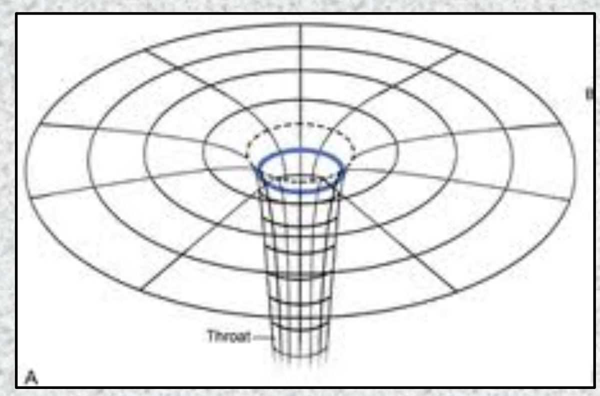
In AdS things tend to return to the origin

Where does AdS come from in AdS/CFT?

AdS tends to show up in certain limits of black hole spacetimes…

Near horizon, infinite throat limit of extremal, charged D=4 black hole is

$$AdS_2 \times S^2$$




Similar situation in Type IIB string theory in D=10

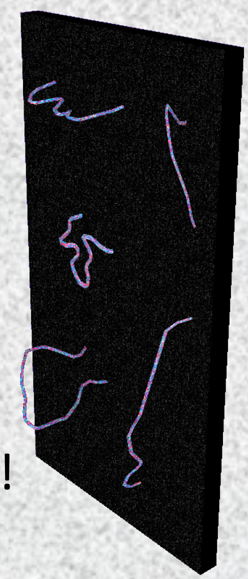
Near horizon, throat limit of D3-brane is

$$AdS_5 \times S^5$$

On the other hand

The low energy dynamics of open strings on the D3-brane is described by

$\mathcal{N} = 4$ Super-Yang-Mills theory



AdS-CFT → These two descriptions are equivalent!

$\mathcal{N} = 4$ SU(N) Super-Yang-Mills theory is a **conformal field theory**

↑ For a stack of N D3-branes

↑ Invariant under global and local rescalings

YM theory in D=4 is classically conformally invariant

Broken by quantum effects – running coupling constant

Field content of maximal Susy theory gives vanishing beta function.

Believed to be a finite theory!

Some key points

- Both $AdS_5$ and $CFT_4$ have the same symmetry group – SO(4,2)
- The CFT is regarded as living at the infinite radius boundary of AdS.
- Roughly speaking it describes the boundary conditions on the AdS box

Precise prescription for AdS/CFT

Witten 1998

- Consider a massless scalar field in $AdS_{D+1}$ $\nabla^2 \phi = 0$
- Bulk field is uniquely specified by behavior on boundary $\phi_0$
- In $CFT_D$ the boundary value of the scalar field couples to an operator of scaling dimension h=D. $\int_{S^D} \phi_0 \mathcal{O}$

Generating function for correlation functions in CFT

$$\left\langle e^{\int_{S^D} \phi_0 \mathcal{O}} \right\rangle_{CFT} = Z_{grav}(\phi_0)$$

Bulk partition function with specified boundary conditions

- Each bulk field couples to a boundary operator in such a fashion

Scaling dimension h related to mass of bulk field

$M^2 = 0$ → h = D marginal

$M^2 > 0$ → H > D irrelevant

$M^2 < 0$ → H < D relevant

OK for negative $M^2$ greater than Breitenlohner-Freedman stability bound couplings & length scales

String theory has overall string length $l_s$ and dimensionless coupling $g_s$

$\mathcal{N} = 4$ SU(N) Super-Yang-Mills theory has dimensionless coupling $g_{YM}$

Related by

$$g_{YM}^2 = g_s$$

AdS$_5$ curvature radius $L = (g_{YM}^2\, N)^{1/4}\, l_s$

t'Hooft coupling is a measure of overall gauge coupling strength

In large N limit with L held fixed and large, string theory is weakly coupled

Classical supergravity is a good approximation

$$Z_{grav}(\phi_0) \simeq e^{-I_{grav}(\phi)}$$

Where do CFT's come from in AdS/CFT?

More general perspective

Also Witten (1998)

To understand global structure of spacetimes in general relativity

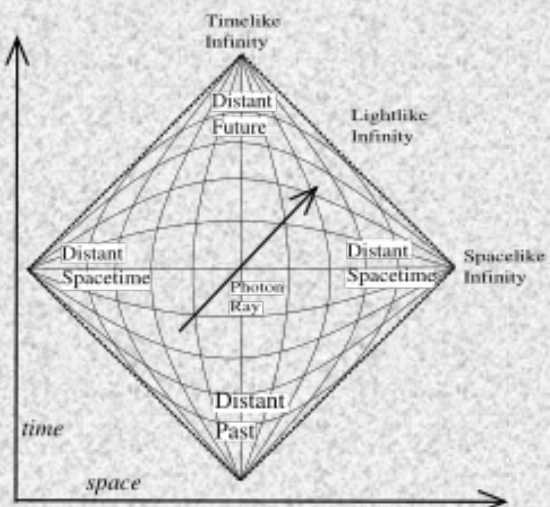
All the way out to infinity

Act with conformal transformation so that metric at infinity is finite

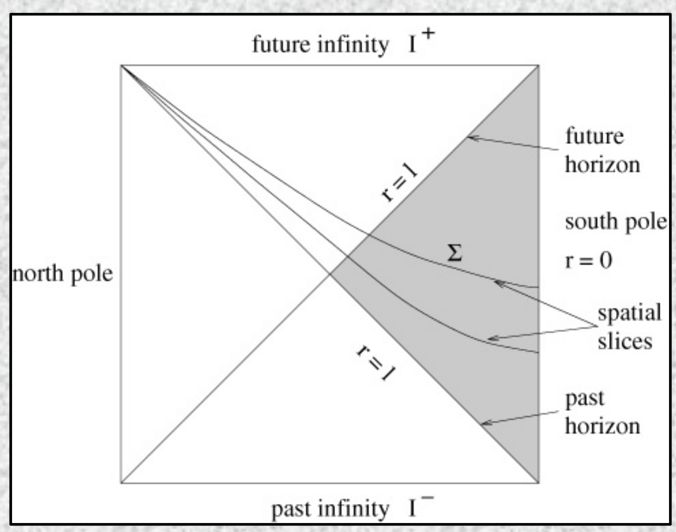
$$\bar{g}_{ab} = f^2 g_{ab}$$

Produce conformal diagrams that include boundary at infinity

Conformal factor has a zero at infinity




Minkowski

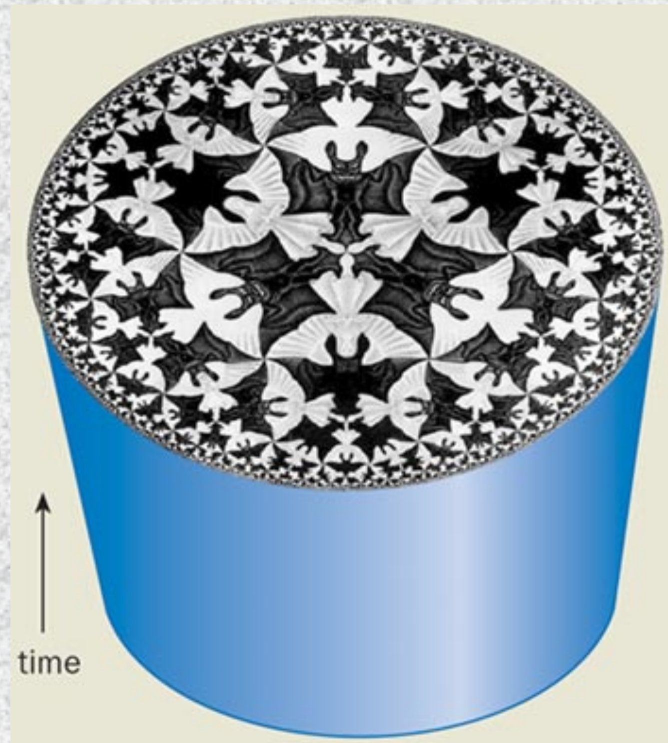



deSitter

AdS$_{n+1}$ conformal diagram

Boundary is $S^n$ x R

time




CFT of AdS/CFT lives on this conformal boundary of AdS

Extending other bulk fields to boundary requires multiplying by powers of conformal factor

Leads to conformal weight of corresponding boundary operator.

| | | |
|---|---|---|
| $M^2 = 0$ → | $h = D$ | marginal |
| $M^2 > 0$ → | $H > D$ | irrelevant |
| $M^2 < 0$ → | $H < D$ | relevant |

Conformal invariance of boundary theory arises because conformal factor can be rescaled by arbitrary function on boundary

$$\bar{g}_{ab} = f^2 g_{ab}$$

Holographic Superconductors

Build up an interesting bulk system

Want to study a phase transition → Need temperature

• Introduce AdS black hole with Hawking temperature T

• In order to capture physics of 2+1 planar superconductors, use planar $AdS_4$ black holes

In asymp. Flat event horizon must be a sphere, but AdS allows more possibilities

• Introduce gauge field, coupled to complex scalar field

• Sufficient to work with "test" matter fields

Ignore backreaction on black hole metric

• Look for low temperature condensate of charged scalar field

Temperature dependent black hole hair

Ruled out for asymptotically flat BH's, but not previously studied for asymptotically AdS

• Try to explore physics of dual, strongly coupled 2+1 QFT

In equations….

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)\,,$$

$$f = \frac{r^2}{L^2} - \frac{M}{r}\,.$$

Planar BH

$$T = \frac{3M^{1/3}}{4\pi L^{4/3}}\,.$$

Hawking Temperature

Note that temperature grows with mass, unlike asymp. flat.

$$\mathcal{L} = -\frac{1}{4}F^{ab}F_{ab} - V(|\Psi|) - |\partial\Psi - iA\Psi|^2\,.$$

$g^{tt}$ negative and large near horizon drives condensate formation

$$V(|\Psi|) = -\frac{2|\Psi|^2}{L^2}\,.$$

Negative mass$^2$ couples to relevant operator in dual field theory.

Look at equations of motion for static, plane symmetric fields

$$\Psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\Psi' + \frac{\Phi^2}{f^2}\Psi + \frac{2}{L^2 f}\Psi = 0\,,$$

$$\Phi'' + \frac{2}{r}\Phi' - \frac{2\Psi^2}{f}\Phi = 0\,,$$

$$A_t = \Phi, \qquad A_r = A_x = A_y = 0$$

2 coupled sceond order ODE's → Start with 4 degrees of freedom

Finite norm for gauge potential at horizon requires $\Phi\big|_{r_H} = 0$ → $\Psi = -3r_0\Psi'/2.$ Removes 2 degrees of freedom.

Near infinity

$$\Psi = \frac{\Psi^{(1)}}{r} + \frac{\Psi^{(2)}}{r^2} + \cdots\,.$$

$$\Phi = \mu - \frac{\rho}{r} + \cdots\,.$$

$\Psi^{(1)}$: Pick to vanish for technical reason…

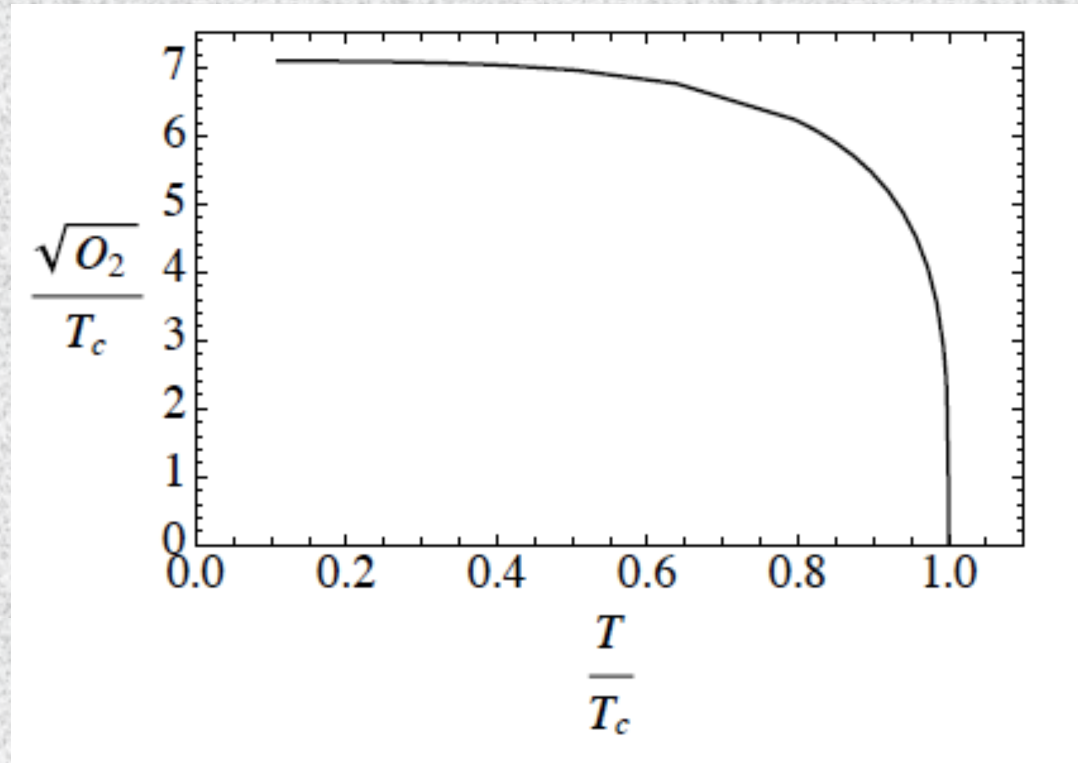
$\Psi^{(2)}$: Measures strength of condensate

$\mu$: Chemical potential

$\rho$: Charge density

Find…..

Charged scalar condensate forms below a critical temperature



Look at response to an electric field along the plane….

DC conductivity diverges below $T_C$

Energy gap develops at lower temperatures.