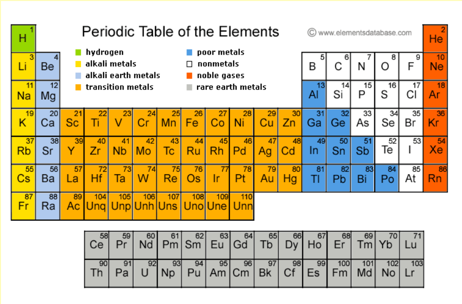
# Black Holes in Higher Dimensions

- Why study the theory of Black Holes?
- Black holes in D=4?
- Why study Higher Dimensions?
- Kaluza-Klein Black Holes in D=5
- Asymptotically flat Black Holes in D=5
- The Ultra-spinning Limit and Blackfolds in D>5

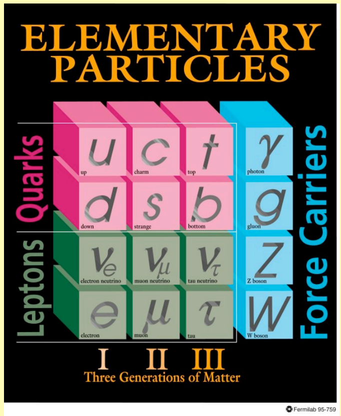
# Why study the theory of Black Holes?




- Black Holes are the "fundamental particles" of gravitation.

  Objects built entirely from the fabric of spacetime.

  Naturally we want to understand everything about their workings.

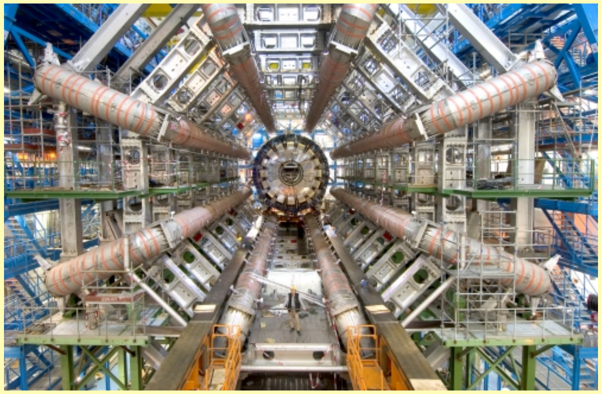



- Black Holes play important roles in the sky.

  Supermassive black holes inhabit the center of most galaxies

  Quasars and Active Galactic Nuclei are among the most energetic objects in the sky.

# Why study the theory of Black Holes?



• The centers of black holes are extreme high curvature environments.

General relativity is expected to give way to a more fundamental theory.

• Black hole thermodynamics

The classical and quantum mechanics of black holes present additional mysteries!

## Black holes in D=4?

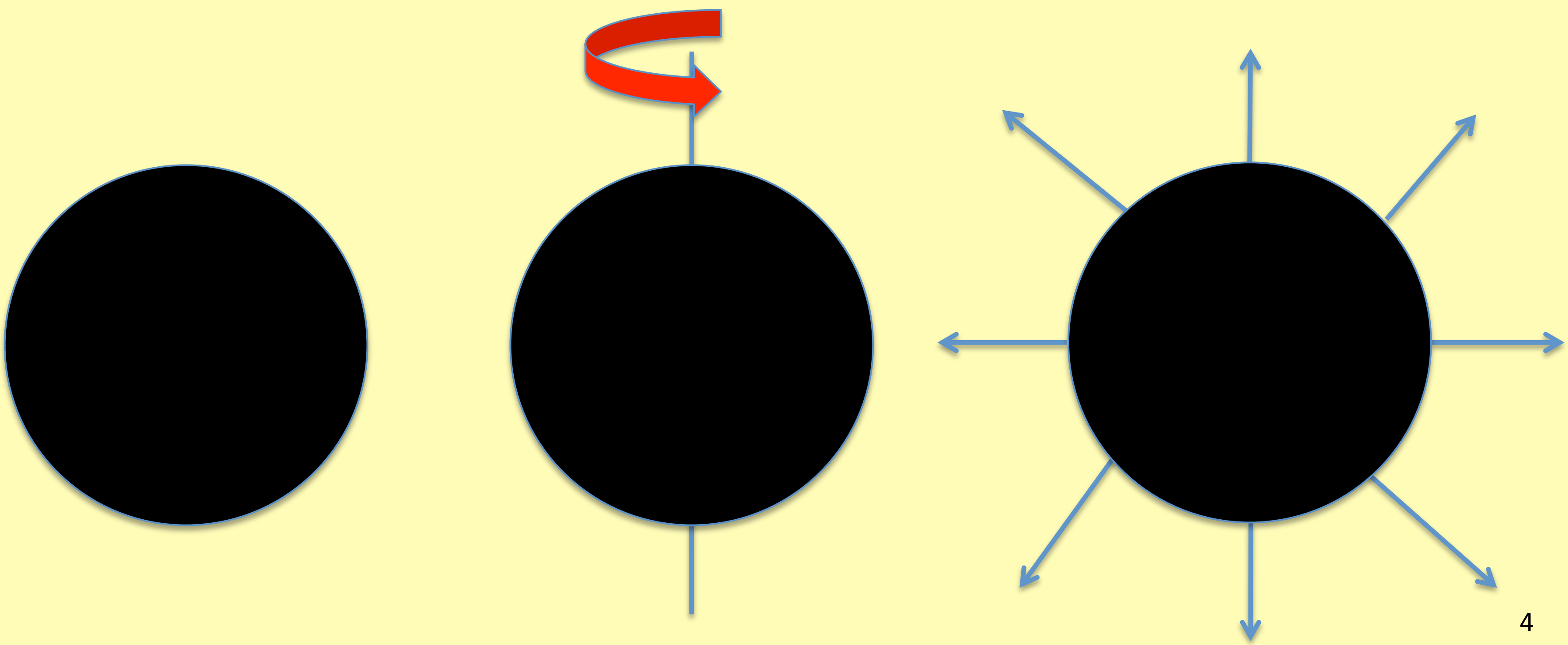
A detailed understanding of D=4 black holes was developed in the 1970's

Focus on "stationary" black holes – no time dependence.

"No Hair Theorems"

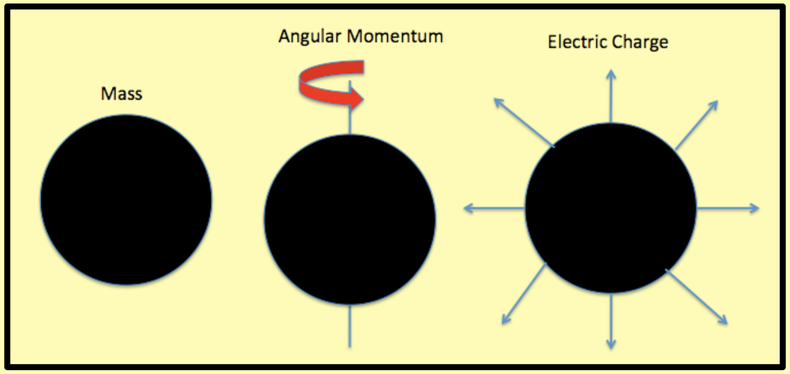
Stationary Black Holes are almost featureless objects

Mass Angular Momentum Electric Charge



# Black holes in D=4?

A Black Hole will settle down into an equilibrium states characterized only by its total mass, angular momentum and electric charge.

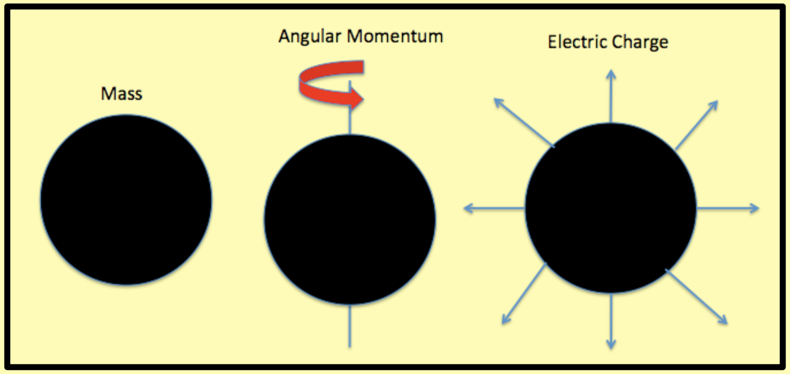



No further trace of what falls in is left behind.

Moreover….

We know the spacetime metric of the most general D=4 equilibrium black hole exactly.

Kerr-Newman spacetime

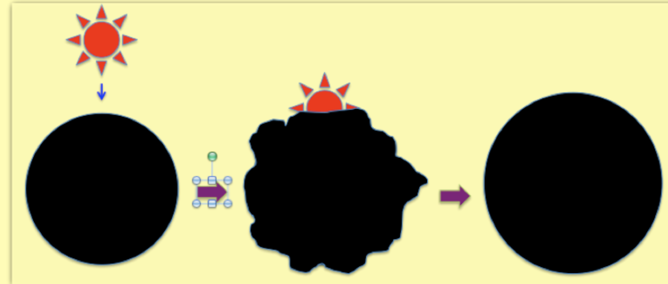



And we can investigate its properties in great detail…..

In D=4 we have indentified our gravitational "fundamental particle" – and fully characterized it in a simple way.

However, this apparent simplicity turns out to be a sign of deeper physics!

**Black holes are like thermodynamic systems….**



Settle down into equilibrium states characterized by a few macroscopic parameters.

Laws of Black Hole Thermodynamics also established in 1970's

Entropy ⟷ Area of Event Horizon

Temperature ⟷ Horizon Surface Gravity

The first and second laws are results in classical general relativity.

$$\delta M = \frac{\kappa}{8\pi G}\delta A$$

$$\delta A \geq 0$$

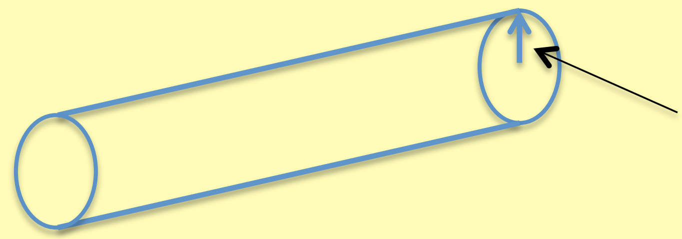
Hawking discovered that quantum mechanics makes black hole actually radiate as blackbodies

$$T = \frac{\kappa}{2\pi}$$

$$S = \frac{A}{4G}$$

Understanding the nature of a black hole's quantum mechanical microstates remains one of the key goals of studies of quantum gravity.

## Why Study Higher Dimensions?




Extra dimensions arise in attempts to achieve of unification of forces with gravity.

Kaluza-Klein theory (1920's) unifies general relativity with Maxwell's E&M by adding a 5$^{th}$ compact dimension.

Mass of electron is far too big

Extra massless scalar field

Nonetheless, Kaluza-Klein theory remains an important theoretical tool.

How are extra dimensions possible?

Don't we know how many dimensions there are from experience?

Fourier expansion → Infinite tower of massive KK states

$$M_n^2 \sim \frac{n^2}{R^2}$$

Particle accelerators rule out (roughly speaking)

$$R > 10^{-19} m$$

## Why Study Higher Dimensions?

String theory requires 6 extra compact dimensions….

The shape of these dimensions determines many properties of 4D physics

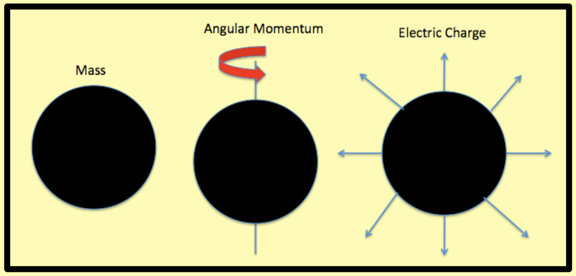
AdS/CFT – the physics of asymptotically AdS spacetimes is related to the physics of quantum field theories in one lower dimension.

At a simpler level…..

The dimension of spacetime is a fundamental aspect of gravity

To understand why the universe is the way it is, we need some appreciation of how it would be different in different dimensions.

## Black Holes in Higher Dimensions?




How are black holes in higher dimensions similar, or different, from D=4?

Still uniquely characterized by mass, charge and angular momentum? ⇨ No — Uniqueness results fail in higher dimensions

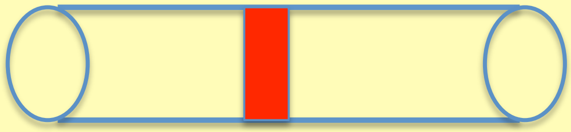
Laws of BH Thermodynamics still hold? ⇨ Yes — Proofs are independent of spacetime dimension.

Consider black holes in D=5
- → Compact 5$^{th}$ dimension
- → Five extended dimensions

One key issue is the shape of the black hole horizon

**Compact 5$^{th}$ dimension** 2 possibilities

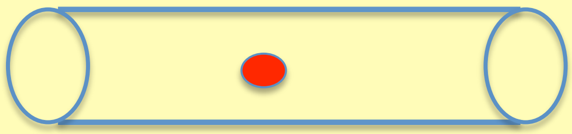
1 - Horizon wraps the compact direction



Simple analytic solution

Uniform black string wrapping compact direction

2 - Horizon localized in compact direction

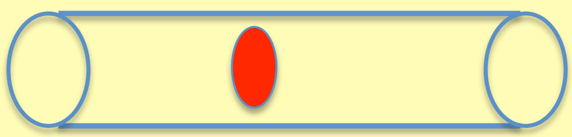


No simple analytic solution

But in the limit of a small BH, expect it to look like Schwarzschild solution

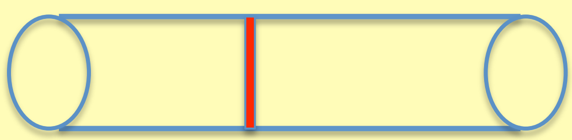
Relations between phases.....

What happens when we increase the size of the black hole?



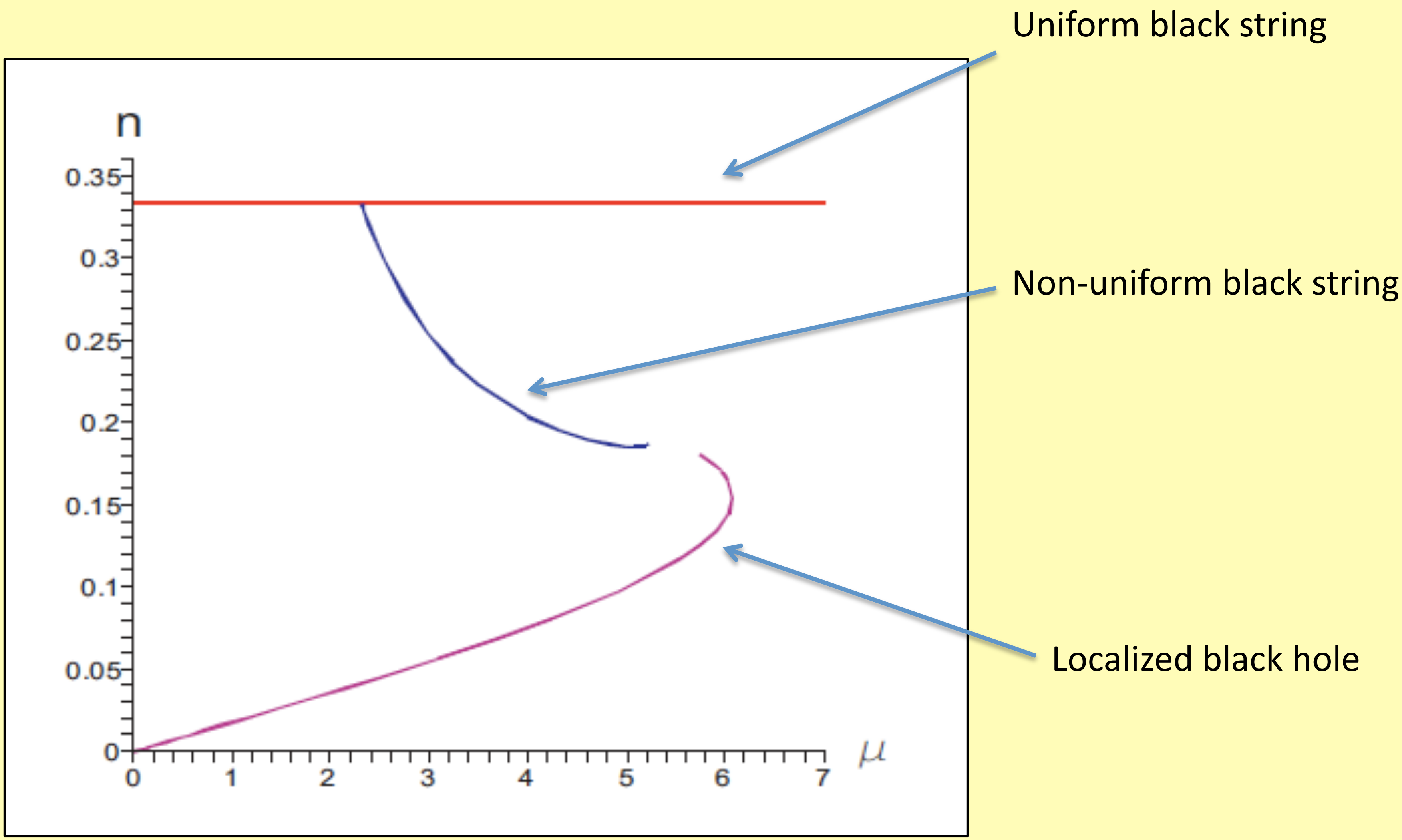
Expect a transition to a wrapped horizon.

Thin wrapped branes are dynamically unstable



Localized black hole has higher entropy.

**Phase diagram for Kaluza-Klein black holes**




From Harmark & Obers – hep-th/0503020

**Extra extended dimension** → Asymptotically flat

Analogue of Schwarzschild black hole — straightforward to write down in any dimension

Analogue of Kerr black hole — Not straightforward, but known

Myers & Perry 1986

(Analogue of Kerr-Newman curiously not known)

D=4 → Can prove that Kerr solution is the most general uncharged, black hole equilibrium state

Key component → event horizon must be a sphere

↑ Not true in higher dimensions!

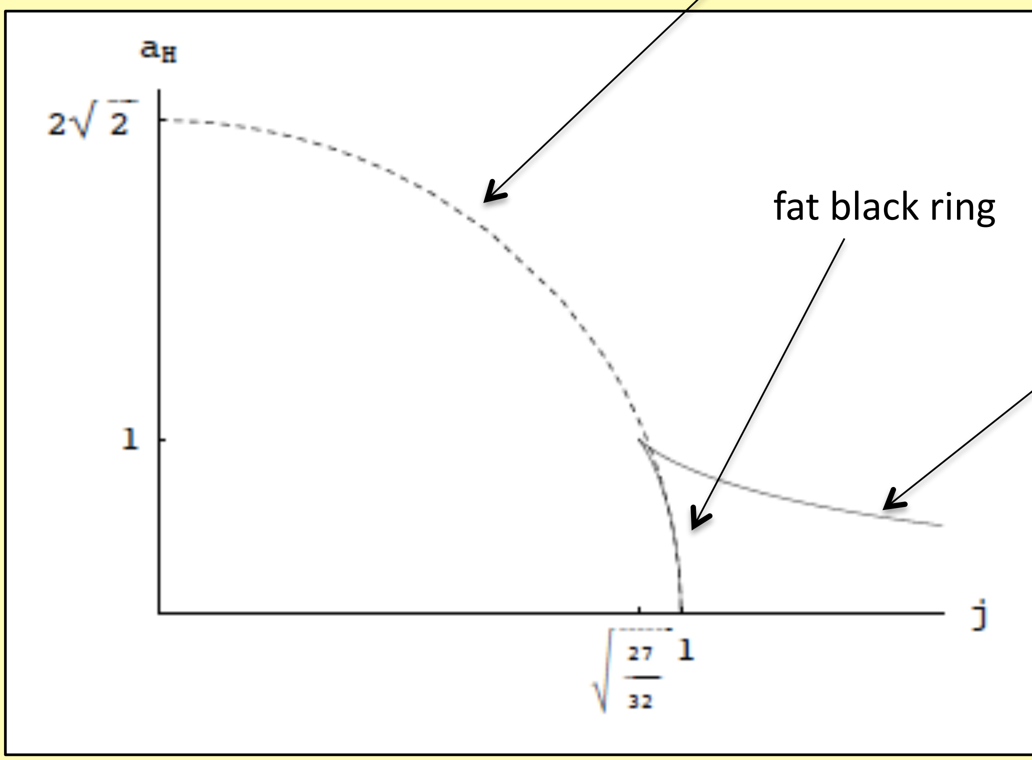
Event horizon must still have positive curvature, but there are more possibilities in higher dimensions

D=5 → Possible event horizon topologies include $S^3$ and $S^2 \times S^1$

**Extra extended dimension**

No examples as yet with more exotic topologies in D=5. → Do they exist?

D=5 rotating Black Ring — horizon topology $S^2 \times S^1$ — Emparan & Reall, 2002




There exists a range of angular momentum with 3 different solutions of equal mass.

No black hole uniqueness in D=5.

**Higher dimensions….**

- Many possible horizon topologies.
- Only known solutions have spherical topology.

No higher D analogue of black ring solution

- In the absence of exact solutions, is there some other form of reliable guidance on which topologies are realized by BH horizons?

Recent proposal by Emparan et. al. that yields approximate solutions → Blackfolds

**World-Volume Effective Theory for Higher-Dimensional Black Holes - arXiv:0902.0427**
**Essentials of Blackfold Dynamics - arXiv:0910.1601**
**New Horizons for Black Holes and Branes - arXiv:0912.2352**

Work in a particular limit in which there are two widely separated length scales. → Ultra-spinning regime

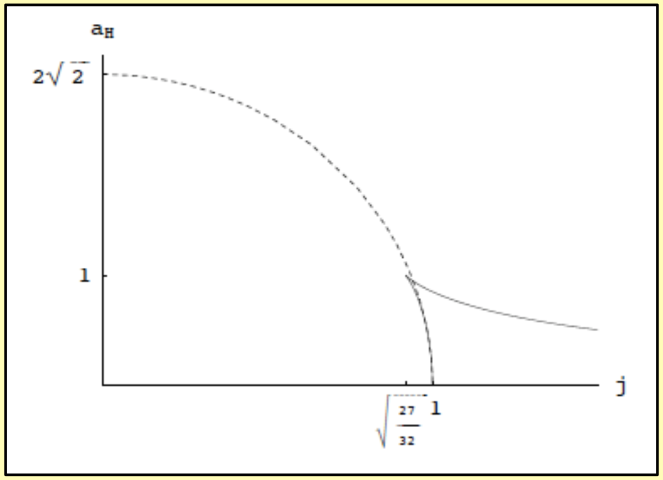
**Ultra-spinning black holes**

D=4 black holes → Maximum angular momentum

Kerr bound

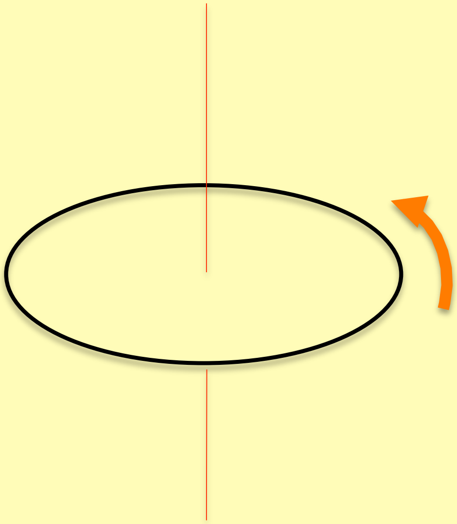
$$J \leq GM^2$$

D=5 black holes




Myers-Perry and fat black rings have bounded angular momentum.

Thin black rings have high angular momentum “ultra-spinnning” regime.

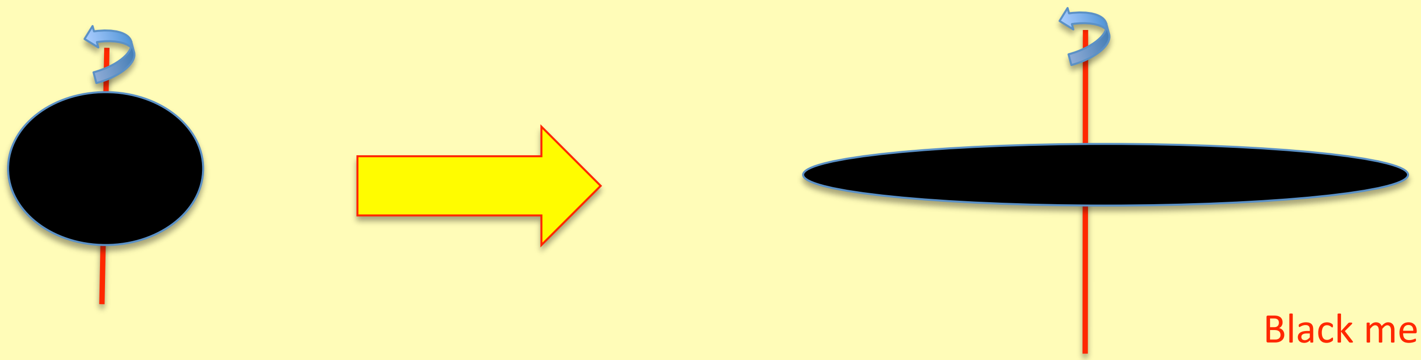


Locally, near the horizon the thin black ring is well approximated by a boosted black string.

**Ultra-spinning black holes**

For D>5 Myers-Perry black holes have an ultra-spinning regime

The horizon looks like a pancake



Near the horizon, the spacetime looks like a boosted black membrane.

Black membranes exist only for D>5, consistent with absence of ultra-spinning black holes in D=4,5.

Conjecture 1 – the near horizon geometry of an ultra-spinning object will approach a boosted black brane.

Conjecture 2 – allowed configurations for ultra-spinning horizons are those that solve the equations of motion for “test” branes.

Solving the blackfold equations of motion has led to a wealth of new horizon configurations.

New configurations for black 1-folds in D>4

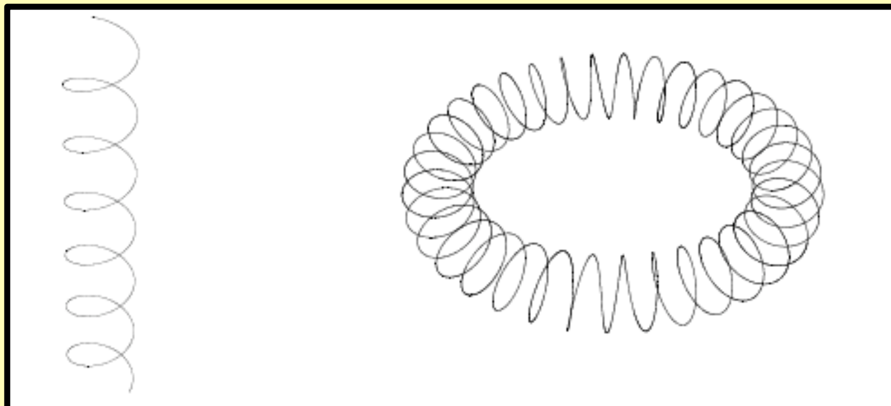


Figure 1: *Helical black strings (left) and helical black rings (right).*

This method gives important clues in one regime, but leaves open…..

- Corresponding solutions to Einstein equations.
- Structure of interior of phase diagram.