Final topics:

Unitarity Methods

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Light Bending

Classical physics from loops

Background Field Method and "t'Hooft-Veltman

ghosts

Non-local Action

Limits to the EFT

Recall

Tree Theorem

This made me investigate the entire subject in great detail to find out what the trouble is. I discovered in the process two things. First, I discovered a number of theorems, which as far as I know are new, which relate closed loop diagrams and diagrams without closed loop diagrams (I shall call the latter diagrams "trees"). The unitarity relation which I have just been describing, is one connection between a closed loop diagram and a tree; but I found a whole lot of other ones, and this gives me more tests on my machinery. So let me just tell you a little bit about this theorem, which gives other rules. It is rather interesting. As a matter of fact, I proved that if you have a diagram with rings in it there are enough theorems altogether, so that you can express any diagram with circuits completely in terms of diagrams with trees and with all momenta for tree diagrams in physically attainable regions and on the mass shell. The demonstration is remarkably easy. There are several ways of demonstrating it: I'll only chose one. Things propagate from one place to another, as I said, with
Unitarity Methods

\[ \frac{\pi}{2} < \text{ on shell} \]

\[ \text{no ghosts!} \]

1) Dispersion relation

\[ \text{Amplitude} = i \frac{\pi}{2} \sum \text{Im} \text{Amplitude} \]

\[ \sum_{\text{loops}} \text{Im} M_{\mu \nu}^{\text{loop}}(p_1, p_2, k_1, k_2) = V \]

\[ p_1 + k_1 + k_2 = p_2 \]

amplitude

\[ P. V. \Rightarrow s_2 I_2 + a_1 I_3 + \ldots \]

recognition by cuts

\[ \Rightarrow \frac{\lambda^4}{16} \frac{c^4}{\ell^2} \]

Universal soft theorem at loop

\[ \Rightarrow \text{Soft theorem: Low, Mack, Gross, Hawking '85} \]

loop also universal

Hilbert, Ross
Double copy

\[ E+M \]

\[ \mathcal{M}_{\text{NNN}}^{(s)}(p_1, p_2, k_1, k_2) = \]

\[ M_{\text{NNN}}^{(s)} = \frac{G^2}{8\pi} \frac{p_1 \cdot p_2 \cdot k_1 \cdot k_2}{k_1 \cdot k_2} \]

\[ \mathcal{M}_{\text{EM}}^{(s)} M_{15}^{(s)} \]

\[ \mathcal{E}_{\gamma^*(x)}(\pm) = \mathcal{E}_{\nu}(\pm) \mathcal{E}_{\gamma^*(x)}(\pm) \]

**Bending of Light**

- amplitude on shell
- multiplicative
- not universal
- not geodesic motion

\[ i\mathcal{M}[\{\gamma^*(k_3)\gamma^*(k_4)\}] = \frac{\kappa^2}{4} \frac{[p_1 \cdot k_1]^2 (p_2 \cdot k_2)^2 (k_3 \cdot k_4)^2}{(p_1 \cdot p_2)(p_1 \cdot k_1)(p_1 \cdot k_2)} \]

\[ \text{classical} \]

\[ \frac{\alpha}{\pi} = \frac{1}{120} \]
Eikonal Method

\[ M(b) = \int \frac{d^3q}{(2\pi)^3} e^{-iqb} M_\text{ren}(q) \]

\[ M(b) = 2(s - M^2) \left( e^{\lambda(b)} - 1 \right) \]

Stationary phase

\[ \theta \approx \frac{4G_N M}{b} + \frac{15 G_N^2 M^2 \pi}{4 b^2} + \left( 8 b n^2 + 9 - 48 \log \frac{b}{2b_0} \right) \frac{\hbar G_N^2 M}{\pi b^3} + \ldots \]

Classical Physics from Quantum Loops

Folk theorem: Loop expansion = \( \frac{1}{b} \) expansion

Not true

Inversi, Gupta-Redfold \( \rightarrow \) classical part

What goes wrong?

At one loop

\[ \hbar \sqrt{\frac{m^2}{\hbar^2}} \sim m_c \]
Background Field Method

$\bar{g} + \theta (g_{\mu \nu} + \kappa h_{\mu \nu})$

Expand around background.

Full covariance w.r.t $g$, $\delta g$

Gauge invariance

$\Lambda \Rightarrow \Lambda = x^\mu \rho^\nu$

$\bar{g} + \theta (g_{\mu \nu} + \kappa h_{\mu \nu})$

$\delta g_{\mu \nu} = \delta g_{\mu \nu} - \delta \bar{g}_{\mu \nu} \bar{\omega}_{\nu} - \delta \bar{g}_{\nu \mu} \bar{\omega}_{\mu} - \frac{1}{3} \delta \bar{g} \bar{\omega} \bar{\omega} + \frac{1}{3} \delta \bar{\omega} \bar{\omega} \bar{\omega}$

$\delta h_{\mu \nu} = \delta h_{\mu \nu} - \bar{D} \delta \bar{\omega} \rightarrow \bar{D} \delta \bar{\omega}$

$K$-covariant
Gross + ghost loops

$$\Delta Z = \frac{1}{16\pi^2} \left[ \frac{1}{2} + \cdots \right] \left\{ \frac{1}{128} R^2 + \frac{7}{128} R_{\mu\nu} R^{\mu\nu} + \cdots \right\}$$

$\xi$ renormal coeff.

Limits of EFT

$$M = M_0 \sqrt{1 + 6g^2 \ln g^2 + 6^2 g^4 + \cdots}$$

$$\text{Fals} \quad 6g^2 \sim O(1) \quad g^2 > M^2$$

$$\text{Fals} \quad 6g^2 > 1$$
IR issue:
Grav. interaction building

\[ \frac{1}{1 - \frac{r}{R}} \]
Large classical effect – Diff.

Only weak field quantities

BHF cascade

Core theory:

\[ Z = \sum_{\{\phi\}} \exp i \int d^4x \left[ -\frac{1}{2} F^2 + e_1 \phi \frac{\partial \phi}{\partial t} + 2 \phi^2 \phi^4 - V(\phi) \right] - \frac{2}{R^2} \phi + e_2 R^2 + e_3 R^4 + \ldots \]

“A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.”

Frank Wilczek
Physics Today
2002