

## Postscript to “Plenitude of Possible Structures” (2016)

The principle of plenitude for possible structures (PPS) that I endorsed tells us what structures are instantiated at possible worlds, but not what structures give the *entire* structure of a possible world, not what *world*-structures there are. A possible structure may be a *substructure* of a world-structure, instantiated by only a subdomain of the domain of inhabitants of a possible world; or it may be a *reduct* of a world-structure, involving only some of the natural properties or relations instantiated at a possible world; or it may be a substructure of a reduct of a world-structure. A possible structure needn't be a world-structure all by itself. For this reason, (PPS) does not provide a complete account of plenitude of worlds when combined with a principle of plenitude for recombinations and a principle of plenitude for world-contents (such as those in “Principles of Plenitude”). For all that (PPS) says, there could be but one (very large!) world-structure, with every world corresponding to some arrangement of *possibilia* within that one structure. In particular, (PPS) will not allow the derivation of various plausible principles of plenitude for world-structures. For example, (PPS) does not tell us whether substructures of world-structures are themselves world-structures, and thus fails to support a principle of solitude according to which any (connected) possible individual can exist all by itself. In this postscript, I first canvas the reasons I had for formulating a principle of plenitude for structures that was noncommittal as to the structure of entire worlds. I then develop a stronger principle that can serve as a principle of plenitude for world-structures in a complete account of plenitude of worlds. The principle I give is strong enough to entail an appropriate version of the principle of solitude, but not so strong as to entail the existence of gunky worlds. Gunk, I am inclined to believe, is impossible.

It might seem, based upon what I say in the introduction to this volume, that I should simply accept:

**(PW1)** Every structure is a possible world-structure.

(This principle is analogous to, but even stronger than, the principle (P1) that I chose not to endorse in the paper.) For I believe that, for every structure, there is a mathematical system of entities that instantiates that structure, entities lacking any qualitative intrinsic nature. And I believe on combinatorial grounds that, for every mathematical system, there are non-mathematical systems instantiating the same structure all of whose entities have qualitative intrinsic natures; maybe they have mass, or charge, or spin. Why not, then, allow that *all*, not just *some*, of these non-mathematical systems are possible worlds, that metaphysical modality quantifies over all of them? Consider, for example, the structures exhibited by simple finite groups, ranging from four-element groups to what is called the “monster” group with nearly  $10^{18}$  elements. I believe that there are systems whose entities instantiate these structures, where all the entities have various masses and charges. But I do not call these systems “worlds”. Why not? Not for any deep reason: these systems exist no less than the systems I take to be “worlds”, and they are no less “concrete.” But calling them “worlds,” and taking them to be metaphysically possible, would depart gratuitously from the common practice of contemporary metaphysicians. My reason, then, is conventional, not ontological. Moreover, I prefer to keep my idiosyncratic ontological proclivities out of the picture as much as possible when communicating my views in the metaphysics of modality.

Perhaps instead I could have simply added to (PPS) that:

**(PW2)** Every possible structure is a possible *world*-structure.

If the “monster” isn’t possible, isn’t instantiated at any possible world – and I know no reason to think that (B) together with (PPS) would entail that it is – then adding (PW2) won’t entail that the “monster” is a world-structure. But (PW2) is still very strong. If a structure is instantiated at a world, then so are all of its reducts and substructures (and substructures of reducts, but I won’t deal with that case separately). Thus (PW2) entails: every reduct or substructure of a possible world-structure is a possible world-structure. When I wrote “Plenitude of Possible Structures,” I did not want to endorse the view that any reduct or substructure of a possible world-structure is itself a possible world-structure. Let me canvas the reasons I had at the time (as well as a new reason) to see which of these reasons hold up and how best to respond to those that do.

I start with *reducts*: structures that are derived by omitting some of the natural properties and relations of the original structure. If we take natural properties and relations to be inhabitants of worlds – immanent universals or tropes – then we might seek to give a Humean justification for the claim that reducts of a world-structure are themselves world-structures: if there are no necessary connections between distinct universals or tropes, then there should be no obstacle to removing any one of them while holding everything else fixed. But extending the Humean project from individuals to properties and relations must be done with some care. I will consider three potential hazards.

First, there is the familiar problem of determinates and determinables: if determinates are natural properties, then it appears that distinct determinates stand in necessary connections. Is it really possible to remove some determinates of a determinable from a world while holding all else fixed? I think not. For example, one cannot remove the mass from an electron without affecting the mass of an atom that contains it. The best Humean solution to the problem, I think,

is to deny that the determinate properties correspond to the natural properties, the universals or tropes. Rather, it is the determinable property that is the natural property, and the different determinates arise from that determinable occupying different places in the property structure at the world. By transferring the necessary features of the determinate/determinable relation to the world's structure, the Humean can continue to maintain that there are no necessary connections between distinct inhabitants of worlds. And by taking the determinables to be the natural properties, there is no obstacle to removing them while holding all else fixed, and taking the resulting reduct of the original world-structure to be a world-structure.<sup>1</sup>

A second and third problem for the Humean approach arises because there are types of structure that are necessary: every world must display structure of that type. Consider first *horizontal* structure: the structure that unites possible individuals into a single possible world.<sup>2</sup> Every world must have horizontal structure. A reduct that omitted the natural relations responsible for horizontal structure, therefore, could not be the structure of any possible world. According to Lewis, the horizontal structure had by all worlds is spatiotemporal structure, structure determined by natural relations that are either spatiotemporal, or analogous to the spatiotemporal relations; worlds are maximal spatiotemporally interrelated sums of individuals. A Lewisian could only accept (PW2) in a restricted form: every *spatiotemporally connected* possible structure is a possible world-structure. A reduct that omitted the spatiotemporal relations, or omitted sufficiently many spatiotemporal relations that the structure fell into disconnected pieces, could not be the structure of any possible world. In "Isolation, Unification, and the Analysis of World" I argued that there are possible worlds whose fundamental structure is not spatiotemporal: the horizontal structure can be provided by any natural external relations;

worlds are maximal externally interrelated sums of individuals. But that still requires the following, weaker restriction to (PW2):

**(PW3)** Every *externally connected* possible structure is a possible world-structure.

Without that restriction, there would be reducts of possible structures that are disconnected world-structures, leading to worlds with disconnected parts. But, one might wonder: don't we want worlds with disconnected parts to represent the possibility of island universes? On my view, however, there is a better way to represent the possibility of island universes.<sup>3</sup> Indeed, if the parts of a world are entirely disconnected and stand in no external relation to one another, in virtue of what can we say that they are parts *of the same world*? I do not think the notion of a world with absolutely disconnected parts is coherent.

The third problem has to do with the type of structure had, not just by all possible worlds, but by *all* systems, mathematical or non-mathematical, in virtue of the principles of the underlying framework. First, there is *identity structure*, the structure determined by the identity relation, the structure in virtue of which the entities in the domain of a system have a definite number. Second, there is *plurality structure*, the structure determined by the is-one-of relation, the structure that grounds plural predication and plural quantification. Third, there is *mereological structure*, the structure in virtue of which the entities are divided into wholes and their parts.<sup>4</sup> Because the relations on which these types of structure are based are present in all systems (if they are present in any), I call them *logical* relations, and the structure they determine, *logical* structure. Now, a reduct of a possible world-structure that omitted a logical relation would not be a possible world-structure. But that is because it would not be a structure at all. I take it to be constitutive of structures that they have these logical features, and that these

features respect the principles of the underlying framework. Thus no further restriction on (PW3) is needed.

I turn now to the question of substructures: is every substructure of a possible world-structure itself a possible world-structure? Consider a world  $w$  with structure  $S$  and let  $D$  be the domain of  $w$ , that is, the class of all parts of  $w$ . Let  $D'$  be a non-empty subdomain of  $D$ . Now restrict all the natural properties and relations that are instantiated by the elements of  $D$  to the elements of  $D'$ .  $D'$  instantiates a substructure of  $S$  in virtue of instantiating those natural properties and relations. But  $D'$  is the domain of part of a world, not an entire world. The question before us is: Is there an entire world whose domain duplicates  $D'$ , and that has as its entire world-structure the substructure instantiated at  $w$  by  $D'$ ? If the answer is “yes”, then we can say that world-structures are preserved under the taking of substructures. We are endorsing what might be called the

**Substructure Principle:** Any substructure of a possible world-structure is a possible world-structure.

Some applications of the principle seem pretty straightforward. If a three-dimensional space is a possible world-structure, then so are two- and one-dimensional spaces that arise from restricting the spatial relations to the elements of a plane or a line. And the principle of solitude to be discussed below is a plausible application. But when I wrote “Plenitude of Possible Structures”, one application seemed to me especially problematic. If one starts with an ordinary Euclidean space composed of points, and considers the subdomain that contains only all the open regions of the space, then the substructure instantiated by that subdomain is gunky: every element of the subdomain has proper parts. (At least, that will be so if we allow that the fundamental relations

apply not only to the points but also to the extended objects, as is commonly done in characterizing topological structure.) But then and now, I was and am inclined to think that gunky worlds are impossible, that the mereological axiom that everything is composed of atoms is necessary and a priori, no less than universal composition and the rest of classical mereology.<sup>5</sup> Thus, I rejected the substructures principle because it appeared to entail the possibility of gunky worlds. I wanted my account of plenitude of structures to be neutral.

But I no longer think this worry is genuine. It is based on a wrong conception of substructure, one that ignores the mereological relations among elements of the original domain. Given that I take mereology to be a part of logic, any subdomain that determines a proper substructure must be closed under mereological operations: for any members of the subdomain, all parts and fusions of those members are also members of the subdomain. Without this condition on substructures, the substructure principle would allow all sorts of worlds that violate the laws of mereology. For example, if one starts with a domain consisting of atoms and all their fusions and considers a subdomain that consists only of the atoms, the resulting substructure violates the mereological law of universal composition, and so fails to be a possible world-structure. Similarly, *if*, as I think, the axiom of Atomicity is a law of mereology, then if one starts with a domain consisting of atoms and all their fusions and considers a subdomain that consists only of composite entities, the resulting substructure fails to respect the laws of mereology and so cannot be a possible world-structure. By requiring substructures to respect the mereological relations of the original domain (no less than other logical relations of the original domain, such as the identity relation), issues about the mereological structure of worlds are kept independent of the substructure principle. An account of plenitude can accept the substructure principle while remaining neutral as to what the laws of mereology are. In particular, the controversy over the

possibility of gunky worlds is removed from the realm of modal plenitude and placed where it belongs, in the realm of logic, broadly construed.<sup>6</sup>

The requirement that substructures must be based on subdomains that respect the laws of logic helps to resolve other problematic applications of the substructures principle. Suppose, for example, that we start with a world whose domain consists of universals and particulars. Restricting to a subdomain containing only universals, or containing universals and particulars that stand in no instantiation relations to one another, would result in a substructure whose domain contains uninstantiated universals, arguably a violation of a framework principle having to do with vertical structure. No worries. Such substructures will be ruled out as illegitimate, and so the Substructures Principle will not entail the existence of worlds with uninstantiated universals. More generally, a theory of plenitude that endorsed the Substructure principle will not come in conflict with any of the framework principles that govern all systems because substructures, as here understood, are required to respect those principles.

Finally, consider the principle of solitude according to which any possible individual can exist all by itself, that is, has a duplicate that is an entire possible world. The substructure principle, it seems, is just what is needed to derive the principle of solitude within a Humean account of plenitude. For, by the substructure principle, for any possible individual  $a$  in a world  $w$  with world-structure  $S$ , the substructure that results from restricting  $S$  to  $a$  is a possible world-structure. And, then, a principle of recombination (such as (LPR), in my “Principles of Plenitude”) will demand that there be a world at which a duplicate of  $a$  instantiates the substructure  $S'$ , and thus is a world onto itself.

But there is a problem. Suppose the possible individual  $a$  is *externally disconnected*: it is the sum of two externally separated parts (where two parts of a world are externally separated if

and only if they are non-overlapping and no part of one stands in any natural external relation to any part of the other).<sup>7</sup> In that case, the substructure that results from restricting the world-structure to *a* will itself be disconnected. But, as already noted above in connection with reducts, no possible world-structure is disconnected lest there be a world with disconnected parts in violation of the demarcation criterion for worlds.

What to do? First, we need to restrict the substructure principle: only externally connected substructures of possible world-structures are themselves possible world-structures. Second, we can then note that this restricted substructure principle only allows the principle of solitude to be derived in a restricted form: for any *externally connected* possible individual, there is a world containing a duplicate of that individual and nothing else. But stopping with that isn't fully satisfactory because the Humean motivation for the principle of solitude applies no less to disconnected individuals. A possible individual, whether connected or disconnected, has an intrinsic nature in virtue of which it can be conceived independently of its surroundings. And if it can be conceived apart from its surroundings, the Humean holds, then, possibly, it can exist all by itself. So we need an unrestricted version of the principle of solitude, one that applies also to disconnected individuals (including transworld individuals). And that is no problem as long as we don't identify *possibly existing* with *existing in some possible world*. If a disconnected individual exists all by itself, the maximal externally related parts of that individual are distinct worlds, not parts of a single world, and the possibility in question is a possibility in which there are island universes. In this way, the principle of solitude holds unrestrictedly when expressed as a claim about what *possibilities* there are, rather than what *possible worlds* there are.<sup>8</sup>

Finally, let us return to the matter at hand: developing an account of plenitude of world-structures strong enough to entail an appropriate version of the principle of solitude, but not so

strong as to entail the possibility of gunk. I suggest that adding (PW3) to (PPS) will do the job. For (PW3) entails the restricted substructure principle: any externally connected substructure of a possible world-structure is itself a possible world-structure. And that in turn entails that, for any externally connected possible individual, there is a world in which a duplicate of that individual exists all by itself. Moreover, by requiring substructures to respect the logical principles of the underlying framework, (PW3) will not entail that there are worlds that deviate from those principles. Disputes over the underlying framework – in particular, disputes over the correct mereological laws – are properly excised from the account of modal plenitude.

## Notes

<sup>1</sup> See my “Is There a Humean Account of Quantities?” (forthcoming) for an extended discussion and defense of this view.

<sup>2</sup> In contrast, *vertical* structure (if any) unites properties and particulars into a single individual; if there are properties and particulars, then the horizontal structure may apply directly to them and only indirectly to the individuals they compose. The examples I used to motivate and illustrate (PPS) all involved horizontal structure; but I did not thereby intend to suggest that that is the only type of structure exhibited by possible worlds.

<sup>3</sup> See my “Island Universes and the Analysis of Modality.”

<sup>4</sup> The fundamental relation by which mereological structure is determined is most often taken to be a parthood relation. I prefer instead to take the fundamental mereological relation to be a generalized identity relation. See my “Composition as a *Kind* of Identity.”

<sup>5</sup> I will not attempt here to argue for the impossibility of gunk. In any case, I have no decisive arguments to offer, nothing that would persuade a gunk enthusiast to change her view. But that isn’t too surprising: I also lack decisive arguments for universal composition. See my “Composition as a *Kind* of Identity” for the best I can do in the latter case.

<sup>6</sup> What about all the mathematical theories of gunky structures if, indeed, gunk is impossible? Mustn’t there be room in logical space for these structures as a legitimate subject matter of mathematics? Indeed; but if gunk is impossible, these structures must be understood abstractly to involve a primitive binary relation between simples called “parthood”, not the genuine parthood relation of mereology. Compare the case of geometrical structure, where points

may either be taken to be parts of line, or instead points and lines may be taken to be simples that stand in a primitive binary relation, *lies-on* (as in Hilbert's axiomatization).

<sup>7</sup> See the discussion of globally vs. locally unified worlds in "Isolation and Unification: the Realist Analysis of Possible World." A world that is locally but not globally unified may contain externally disconnected individuals.

<sup>8</sup> This solution requires that one accept absolute actuality. See my "Island Universes and the Analysis of Modality" where I use the principle of solitude applied to disconnected individuals to argue for the possibility of island universes.