

ON LIVING FOREVER

Phillip Bricker

I want to live forever; but just what is it that I thereby want? Prior to 1874 (or thereabouts) my want would have seemed quite clear: I would have wanted to live for an unending sequence of years, one year for each natural number--an omega-sequence of years. But our horizon has since been expanded by the teachings of Georg Cantor.¹ The natural numbers all together amount only to the smallest order of infinity, aleph-null. There are countless greater infinities that dwarf aleph-null as surely as aleph-null dwarfs our customarily allotted three score and ten. Why settle for a piddling aleph-null years if there are limit cardinals out there to vault over, inaccessible cardinals waiting to be surpassed?

This paper divides into three parts. In the first part, I argue that trans-omega longevity is (conceptually) possible: there are possible worlds that endure beyond a single omega-sequence of years, and a person can survive in these worlds from one omega-sequence to another. In the second part, I discuss two reasons why one might want to live for more than a single omega-sequence, one having to do with the pursuit of mathematical knowledge, the other with the maximization of pleasurable experience. Finally, in the third part, I provide an analysis of wanting to live forever, in the sense of wanting to live as long as possible. Is it wanting to inhabit a world in which one's life has the maximal possible duration? I argue

that there are no such worlds, and then provide an alternative analysis.

I want trans-omega longevity, but not at any cost. Wanting to live beyond a single omega-sequence of years is, for me, a conditional want, as is wanting to live to be 100. Both wants are conditional, at the very least, upon my still having my wits about me, and upon there still being a fair balance of pleasure over pain. In claiming that trans-omega longevity is desirable, I claim only that there is some possible world, even if quite remote from our own, in which I have trans-omega existence and the above conditions are satisfied. Some, it is true, have argued that such conditions could never be satisfied even for ordinary immortality because a life too long inevitably leads to perpetual boredom.² I suspect that those who argue in this way either lack imagination or become too quickly jaded with the good things in life; at any rate, their arguments do not, so far as I can introspect, apply to me. I will not attempt to respond directly to their arguments here. Instead, I will assume that immortality of one sort or another is desirable, and ask what kind of immortality I would choose, if I had the choice.

I

One objection to the claim that I want to live for more than aleph-null years needs to be dismissed at the outset. What if the actual world has at most aleph-null years to give? Would it follow that I could not want to live longer than that? No, the life span of

the actual world sets no limit upon what I can want. I could want to live aleph-null years even if the universe were destined to end in the Big Crunch some few billions of years down the line. Similarly, I can want to live for more than aleph-null years whether or not the universe actually has those years to give. My wants are not circumscribed by what is actual, only by what is possible.³

Still, one might wonder whether it is even possible for a world to endure for more than aleph-null years. In answering this question, I will focus upon those worlds that can be decomposed into a temporally ordered aggregate of instantaneous world-stages, and that support a (possibly transfinite) metric that measures time intervals between stages in years. Moreover, I will assume that each world-stage is contained within an omega-sequence of years, where an omega-sequence of years is itself an aggregate of world-stages that is isomorphic to the half real line. Finally, the world as a whole is a well-ordered aggregate of these omega-sequences.⁴ A world with only one omega-sequence of years I will call an ordinary world. I will say, somewhat metaphorically, that stages from neighboring omega-sequences are separated by a gap. More exactly, two stages are separated by a gap if and only if the interval between them as measured by the temporal metric is infinite. Cantor demonstrated the abstract possibility of temporal orderings that allow world-stages to sum to worlds that endure for more than aleph-null years. But there are two sticking points: there is a problem of quantity, of there being enough world-stages to make such a world; and there is a problem of unification, of uniting the world-stages into a single world. I will take up the second problem first.

A world that endures for more than aleph-null years would have to have an omega-sequence of years followed by more years. On what grounds would the further years be accounted part of the very same world? Wouldn't a mysterious world-building glue have to be posited to hold such a world together across the gap?

Mysterious, perhaps; but no more mysterious than what is already needed to keep an ordinary world from falling apart at its Dedekind seams. One way of arguing for this invokes a modal principle of the elasticity of time:

Elasticity Principle. Any sequence of events might have occurred faster or slower without affecting any non-temporal relations between the world-stages comprising the world.

Consider the sequence of events that will occur between 11:00 PM and midnight on the eve of the year 2000. There is a possible world in which the sequence from 11:00 to 11:30 takes one year to occur, in which the sequence from 11:30 to 11:45 also takes one year to occur, and so on ad infinitum. In this elongated world the events prior to the year 2000 are stretched out over an omega-sequence of years; the events of midnight occur not at the beginning of the third millenium, but at the beginning of a second omega-sequence. Whatever (non-temporal) relations serve to hold together the world-stages before and after midnight in the ordinary world are still around to keep the elongated world from falling to pieces. So the unification of worlds composed of successive omega-sequences of years is in general no more problematic than the unification of an ordinary world.

The argument assumes, of course, that the temporal relations do not themselves supply the glue; else the world might come unglued when the temporal relations are tampered with. This assumption is unproblematic for those (like myself) who accept the irreducibility of temporal relations. How do things stand for the reductionist with respect to time? The reductionist believes: No time without a timekeeper! The timekeeper might take many forms: it might be a transcendent being that establishes temporal relations between stages by its subjective perception of them; it might be a clock ticking away in some corner of an otherwise uneventful universe; normally it will be a convergent pattern of cyclical, lawful processes from which a temporal metric can be defined. The reductionist can accept the elasticity principle in the amended form:

Amended Elasticity Principle. Any sequence of events might have occurred faster or slower without affecting any relations between world-stages other than relations involving the timekeeper.

The argument still goes through if the reductionist believes there is a world in which the timekeeper is sufficiently localized or isolated that it is implausible to take the timekeeper to be what holds the world together; the amended elasticity principle can then be applied to this world. Reductionists with an essentially sticky conception of time may remain unconvinced by the argument; but I suspect there are other paths to the same conclusion.

If two omega-sequences of years can be brought together to form a single world, I see no reason why the same would not hold for a

larger number of omega-sequences, even infinitely many. That brings us to the problem of quantity. Repeated application of the elasticity principle leads to more and more complicated temporal orderings, but never turns an ordinary world into a world with more than aleph-null years. To ensure the existence of such worlds, we can use the following modal principle of plenitude:

Principle of Duplication. Whatever can be duplicated can be duplicated in any quantity whatsoever.

This principle follows from the more general modal maxim: possibility respects no arbitrary bounds. There may be in the actual world no more than seventeen duplicates of some object, say, some kind of quark; but that couldn't be a necessary truth. The same goes for aleph-seventeen.

Not everything that exists can be duplicated. Consider the number three. It is as much an absurdity to speak of many threes as to speak of a round square. The distinction between those objects that cannot by their nature be duplicated and those objects that can be duplicated corresponds roughly with the traditional distinction between abstract and concrete; indeed, it provides one way of explicating that distinction.

Among the entities that can be duplicated are world-stages; otherwise there could be no world at which time passes without anything happening.⁵ So the duplication principle ensures that there are enough world-stages around to make a world with more than aleph-

null years, indeed, with as many years as one pleases. Such worlds might be exceedingly dull, for all the principle tells us, but they are possible nonetheless.

I have argued that a single world can encompass any number of omega-sequences of years, and can hold together across the gaps between them. But I need more. I need to argue that a person inhabiting such a world can survive the gaps. What relations must hold between person-stages on either side of a gap for them to be stages of one and the same person? I answer, with many others, that they need to belong to an aggregate of person-stages that exhibits the right sort of psychological continuity and connectedness.⁶ Now, the elongated world introduced above already demonstrates the possibility of surviving the gap: whatever relations of continuity and connectedness hold between the stages of an ordinary person whose life straddles the year 2000 in the ordinary world also hold between the stages of his elongated counterpart whose life straddles the gap in the elongated world. But this is longevity without the benefits. The life and accomplishments of the elongated person are quite ordinary in all non-temporal respects. Is there a more interesting way to survive the gap?

First, let us consider the problem of continuity. The problem is that complete continuity at the limit point initiating a second omega-sequence of years would require that all changes in one's psychological makeup became vanishingly small as the preceding omega-sequence of years progressed. And that seems to undercut the

rationale for wanting to live more than one omega-sequence of years because it leaves no room for personal growth. The problem evaporates, however, as soon as one realizes that continuity need not be absolute for identity to be preserved. Say that my interest in trans-omega longevity stems from an interest in the transfinite accumulation of knowledge. My surviving the gap would no more be threatened by a constant increase in knowledge than would my ordinary survival be threatened by the ingestion of a Britannica pill, a pill that furnishes instant encyclopedic knowledge. Survival only requires that there be continuity in enough of the psychological traits that matter. And, certainly, in wanting to live beyond a single omega-sequence of years, I do want most of my important character traits and values to stay with me. So, on a reasonable criterion of psychological continuity, my wants are not incompatible with surviving the gap.

What about psychological connectedness? The relation of psychological connectedness generally held to be most important to personal survival is the relation that holds between two person-stages whenever one contains, or under appropriate conditions would contain, a memory of an experience had by the other. I see no obstacle to remembering experiences from a previous omega-sequence. True, facts of brain physiology severely restrict the time periods over which direct connections of memory can persist in the actual world. But these facts are contingent, and thus do not preclude the possibility of surviving for longer periods, even omega and beyond. They only show that in wanting to live for omega years or beyond, I

also want there to be a more reliable mechanism for encoding memories.

But there is another problem. It is arguably essential to us as persons that our minds be finite, and, in particular, that at any moment we can remember at most a finite number of distinct experiences from our past. Thus, at the onset of a new omega-sequence of years, all but finitely many of the previous years will be lost to oblivion. This might indeed sour some to the prospects of trans-omega living, namely, those who cherish their experiences more in the remembering than at the time. But I don't think it is an obstacle to surviving the gap between omega-sequences. It is still possible for any two person-stages to be connected by the transitive closure of the memory relation. That is all we ask for in ordinary cases of survival; why demand more when trans-omega survival is at stake?⁷

II

I have argued that trans-omega existence is within the realm of possibility, and thus a legitimate object of desire. But I can hear the reader impatiently demand: "What could be the point of it? What could one do or experience in more than one omega-sequence of years that could not already be done or experienced by an ordinary immortal living for one omega-sequence?" I will consider two replies. The first reply invokes lofty intellectual pursuits, but is somewhat inconclusive. The second reply invokes more creaturely pleasures and

would equally be available to a slug. I think it succeeds, however, in giving me reason to want trans-omega longevity.

Consider first my interest in mathmematics. For example, I have always wanted to know whether Fermat actually possessed "a truly marvelous proof that would not fit in the margin." Is Fermat's Last Theorem true? Trans-omega longevity would give me the means to find out. And I wouldn't even need a Ph.D in mathematics; just the patience to perform trivial, though increasingly tedious calculations.⁸ I could simply begin a systematic search for counterexamples, deciding in advance to write the word "no" in some designated place when and only when a counterexample was found. Then, assuming a world in which inscriptions survive the gap, the answer will be waiting for me at the beginning of the next omega-sequence: if I find the word "no", the conjecture is false; otherwise, true.

There might appear to be a catch in the above procedure in light of what was said about the essential finiteness of memory. I do not have in mind the fact that I would be able to recollect performing at most finitely many of the calculations necessary to a verification of the conjecture; the rest is mercifully forgotten. But the procedure does require that I have some way of knowing when the next omega-sequence has arrived so that I can know whether to interpret the lack of a "no" as a "yes". Can I know that a new omega-sequence has arrived without remembering the intervening years? I think so. The fact that I no longer intend to perform another tedious calculation

gives evidence of such. Moreover, the world might contain an ideal clock so designed that its hands move half the distance to twelve o'clock with each passing year; the chimes at midnight then signal the arrival of the new epoch. Memory is not essential to marking the passage of time, either in a world of ordinary or of trans-omega duration.

Why, then, do I find this first reply inconclusive? Because there are other sure ways of determining the truth or falsity of Fermat's Last Theorem that would not involve so much waiting around. My interest in Fermat's Last Theorem doesn't so much give me reason to want trans-omega longevity as reason to want to inhabit a world in which infinity machines exist, machines that can perform an omega-sequence of tasks in a finite amount of time.⁹ If my interest in acquiring mathematical knowledge is to give me a reason to want trans-omega longevity, there will have to be mathematical problems that are solvable given enough time, but not solvable by an infinity machine in an ordinary world.

Here one naturally turns to undecidable problems in set theory. Solutions to some of these problems would be worth waiting around for no matter how long the wait. Unfortunately, there seems to be no systematic way to inspect the universe of sets, one set at a time, analogous to the way it is possible to inspect the universe of numbers. The problem has to do with the non-constructive character of the cumulative hierarchy, and is not at all affected by merely adding more time.

Consider the case of the Continuum Hypothesis. Its truth or falsity is already decided by the time the cumulative hierarchy has been carried out two levels past level omega. If one could inspect all of the sets at this level, one could decide the Continuum Hypothesis simply by checking to see whether, for every set of reals, there exists (a set that is) a one-to-one correspondence between it and either the set of all the reals or an enumerable set of reals. But it is not so simple. Ignore for now the problem that the requisite bookkeeping could not be done within the confines of an ordinary Euclidean space; perhaps we could expand our spatial horizon just as we have expanded our temporal horizon. The insurmountable problem is that we would have no way of knowing that we had inspected all the sets of reals, or all the one-to-one correspondences between them.

It seems doubtful that transfinite living would do much for the pursuit of mathematical knowledge that could not also be done by other means.¹⁰ But there is another sort of reply to the question: what point could there be to living for more than a single omega-sequence of years? And if it is right, it equally gives reason for wanting to live for any number of years, no matter how large the number. It begins by noting that there are pleasures that we never grow tired of, of which we can say: "the more, the better." Take, for example, my unflagging desire for Thai food: I want to eat it this week, and next week, and the week after, and so on. Do I thus desire more than aleph-null experiences of Thai food?

There is a problem owing to an ambiguity in the phrase 'and so on'. Do I really want my Thai experiences to be iterated far into the transfinite? Or do I want only that every Thai experience be followed by another? The latter want, of course, could be satisfied within the lifetime of an ordinary immortal. To the extent that it is unclear which of these wants I have, it is also unclear whether my enjoyment of Thai food gives me reason to want to live for longer than an ordinary immortal. And indeed, although I am quite sure that I never want to be eating my last Thai dinner, I am less sure what attitude I have towards the prospect of future omega-sequences without Thai food.

The ambiguity occurring in 'and so on' occurs in exactly the same way in 'forever', and that takes us back to where we began. In wanting to live forever, do I want only that the sequence of years comprising my life have the ordinal property of having no last member? Or do I also want that the number of years of my life have the cardinal property of being as large as possible? Only in post-Cantorian times has it become clear that the former property fails to entail the latter.

There is no doubt that I have the ordinal want. It comes from viewing my life from within and realizing that I never want to be at the end of my life, that I always want to have more to look forward to. But here, in contrast to the Thai food example, I feel quite sure that I also have the cardinal want.¹¹ It comes from viewing my life from without and realizing that the longer life is the better

life provided only that the added years are themselves worth living. And, I claim, there always is a longer life: for any of my possible lives in any possible world, there is another possible world in which that life is (cardinally) extended by adding years that I deem worth living. That there should always be such worlds follows from the principle of duplication, and the fact that I am easy to please. For one of the pleasures that I will never grow tired of is life itself; at any rate, life spent in a thinking, active mode, free of pain.

III

One doubt may remain. I have claimed that in wanting to live forever, I want the number of years of my life to be as large as possible. But how large is that? Since the cardinal numbers increase without bound, there is no particular number of years that I want to live. It appears, rather, that I want the following to be the case: for any particular number of years, I live longer than that. But, assuming that I must live for some particular number of years, the above want isn't satisfiable in any world; in wanting to live forever, I would be wanting the impossible.

One solution would be to give up the assumption that I must live for some particular number of years. One could posit the existence of worlds that endure for longer than any cardinal number of years, worlds, for example, whose omega-sequences are ordered like the class of all ordinal numbers. Such worlds might be called ageless because no numerical age can be assigned to them. On this view, wanting to live forever is wanting to be ageless in an ageless world.

I prefer not to posit ageless worlds, although I have no decisive objection to them. I cannot object on the grounds that they bring with them a prima facie commitment to proper classes. True, the domain of individuals existing at an ageless world is "too large" to form a set, since there are as many world-stages as there are ordinal numbers, and each world-stage is itself an individual existing at the world. But I am already committed to the totality of worlds being "too large" to form a set because, I have claimed, for every cardinal number there is a world that endures for exactly that many years. I find proper classes obscure and expect they will find no place in a final inventory of what there is. But I know of no reason to think that positing ageless worlds is any worse than positing arbitrarily long-lasting worlds on the score of proper classes.¹² In either case, I believe, a Zermelo-style solution could be brought to bear to eliminate the problematic classes.

Still, I think there is reason enough to reject ageless worlds. They conflict with ordinary modal intuitions about time and persistence. The idea that something might endure, but not endure for any particular length of time, sounds, on the face of it, absurd. And further modal reflection does nothing to overturn the initial impression. In particular, principles of plenitude such as the principle of duplication do not require the existence of ageless worlds, at least not for a non-believer in proper classes. The principle of ^{duplication}~~plenitude~~ requires that, for any object that can be duplicated and any cardinal number, there is a world containing just that number of such duplicates. It thus requires that the totality

of worlds be "too large" to form a set. But it does not require that the domain of individuals at a single world ever be "too large" to form a set. Transfinite longevity notwithstanding, our days are necessarily numbered.

Nor are ageless worlds required to provide an analysis of wanting to live forever. To see this, let us first consider a structurally analogous case that admits of a similar solution. I once knew a person who, I would say, had a desire to memorize as many digits in the decimal expansion of pi as possible. How might this desire be analyzed? Did she want the following to be the case: for any (finite) number, she memorized more than that many digits in the decimal expansion of pi? If we assume what was said above about the essential finiteness of memory, that would be to want the impossible. Is there some way to explain what she wanted without attributing to her a confused desire for the impossible? Indeed, what she had was an infinite sequence of distinct wants: for each (finite) number, she wanted to memorize more than that many digits in the decimal expansion of pi. Each want in this infinite sequence is satisfied in some possible world. What is impossible is only that all of these wants be simultaneously satisfied.¹³

This solution transfers easily to the case at hand: providing an analysis of wanting to live forever, in the sense of wanting to live as long as possible. Although there is no world in which I could be said to live forever, it is possible to understand what I assert when I say: I want to live forever. There is no unitary want that I

have, but instead a transfinite sequence of wants, each satisfiable: for any cardinal number of years, I want to live longer than that. Or, even better, think in terms of a personal utility function defined over the worlds: in my case, worlds in which I live longer in general score higher, but without there being a highest-scoring world.¹⁴ This shows that, by my lights, there is no best of all possible worlds: they get better and better as I live longer and longer. Pace Leibniz, this world barely rates at all.

Yale University

FOOTNOTES

1. In 1874 Cantor first published a proof that the real numbers are not equinumerous with the natural numbers. See "Über eine Eigenschaft des Inbegriffes allen reellen algebraischen Zahlen," Gesammelte Abhandlungen (Berlin: Springer Verlag, 1932).
2. See, for example, Bernard Williams, "The Makrapulos Case: Reflections on the Tedium of Immortality," in Problems of the Self (Cambridge: Cambridge University Press, 1973), pp. 82-100.
3. Are my wants even circumscribed by what is possible? I claim that one cannot genuinely want what is impossible; but this claim is controversial, and I cannot here discuss how I would deal with the various prima facie counterexamples. Very little will depend upon this claim in what follows.
4. It is for convenience only that I restrict my attention to this class of worlds. I do not mean to suggest that there are no worlds outside of this class: worlds with no time at all, or time but no temporal metric; relativistic worlds, or other worlds that cannot be decomposed into temporal parts; worlds with no beginning, or with a beginning and an end; worlds whose omega-sequences are not well-ordered; and so on.
5. If an argument is wanted for the possibility of time without change, see Sydney Shoemaker, "Time Without Change," Journal of Philosophy, 66 (1969), pp. 363-381.

6. Locke is generally held to be the father of this view. Modern exponents include Derek Parfit, John Perry, and David Lewis. See Derek Parfit, "Personal Identity," The Philosophical Review 80 (1971), pp. 3-27; John Perry, "The Importance of Being Identical," in A. Rorty, ed., The Identities of Persons (Berkeley: University of California Press, 1976), pp. 67-90; and David Lewis, "Survival and Identity", in A. Rorty, op. cit., pp. 17-40.
7. The above problem is like the familiar case of the senile general, but with a vengeance. For the solution in the ordinary case, see John Perry, "The Problem of Personal Identity," in John Perry, ed., Personal Identity (Berkeley: University of California Press, 1975), especially p. 19.
8. Of course, the enterprise need not monopolize all of my time: I could arrange it so that the time spent calculating took up an increasingly smaller proportion of my total time; or a trustworthy computer could be employed.
9. For more on infinity machines, see the essays by Black, Thompson, Benacerraf, and Grunbaum in Wesley Salmon, ed., Zeno's Paradoxes (Indianapolis: Bobbs-Merrill, 1970).
10. But I here leave open the question whether any interesting mathematical proposition could be decided in this way. Perhaps each level of the constructible universe L could be exhaustively inspected, one set at a time, so as to decide sentences

undecidable in the theory $ZF + V=L$. Perhaps attempts to produce infinitary proofs would improve upon what infinity machines can prove in an ordinary world.

11. Here, of course, I can only speak for myself. I do not claim that one who lacks this want is irrational.
12. Terrence Parsons has influenced my thinking on this point.
13. Of course, this solution only works if one need not want the conjunction of what one separately wants, lest one be back to wanting the impossible. But this is familiar behavior of wants-that as a propositional operator.
14. It cannot be a real-valued utility function. There aren't enough real numbers to make the necessary discriminations.